

TRAP

ATTENUATORS

and if  $Z_1 = Z_2$ ,

$$R_1 = R_2 = Z$$

$$R_3 = \frac{2Z_1 \sqrt{K-1}}{K-1}$$

For the balanced and unbalanced  $\pi$  networks the values of  $R_1$ ,  $R_2$ , and  $R_3$  can be determined as follows:

( $\pi$ )

$$R_1 = \frac{K}{K+1} Z_1$$

$$R_2 = \frac{K}{K+1} Z_2$$

$$R_3 = \frac{K-1}{2} Z_1$$

and if  $Z_1 = Z_2$ ,

$$R_1 = R_2 = Z_1$$

$$R_3 = \frac{Z_1(K-1)}{2\sqrt{K}}$$

Example 17.1

Design a network to match a 500-ohm generator to a 200-ohm load with a minimum possible power loss.

Solution

1. Determine the ratio of  $Z_1$  to  $Z_2$  and the minimum possible value of the power ratio  $K$ .

$$\frac{Z_1}{Z_2} = \frac{500}{200} = 2.5$$

From Eq. (17.1)

$$K_{min} = 2 \times 2.5^2 - 1 = 7.87$$

2. Determine the type of network to be used. Since the type of network was not specified, it was stated that the network loss shown in Fig. 17.3 it is stated that  $R_2 = 0$  ohms when  $Z_1 \geq Z_2$  it is only necessary to determine  $R_1$  and  $R_3$ . From Eqs. (17.2) and (17.3)

$$R_1 = \frac{500(7.87 + 1) - 2\sqrt{7.87 \times 500 \times 200}}{7.87 - 1} = 387 \text{ ohms}$$

$$R_3 = \frac{2\sqrt{7.87 \times 500 \times 200}}{7.87 - 1} = 258 \text{ ohms}$$

(Refer to Fig. 17.4.)

3. Determine the loss in the network.

$$\text{Loss} = 1 - \frac{1}{K} = 1 - \frac{1}{7.87} = 0.87$$

17.1. Fixed Attenuators. Networks which introduce a fixed amount of attenuation independent of frequency have extensive use. They can be designed to have

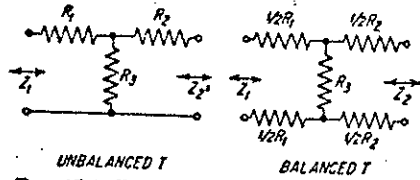


Fig. 17.1. T and H attenuator networks.

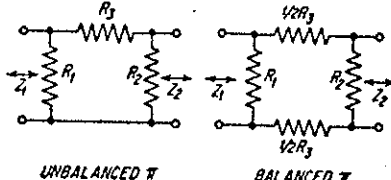


Fig. 17.2.  $\pi$  and O attenuator networks.

equal or unequal input and output impedances and to provide different amounts of attenuation.

Unbalanced and balanced T and  $\pi$  networks are shown in Figs. 17.1 and 17.2. For every ratio  $Z_1/Z_2$  of the values of terminating impedances there is an associated minimum value of the ratio of input power to the attenuator to output power from the attenuator which can be realized.

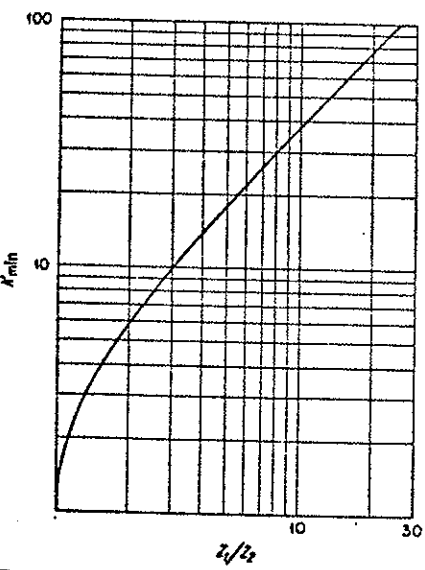


Fig. 17.3. Plot of the minimum possible values of  $K$  as a function of  $Z_1/Z_2$  (refer to Figs. 17.1 and 17.2). If  $K = K_{min}$ ,  $R_1 = 0$  in Fig. 17.1 and  $R_1 = \infty$  in Fig. 17.2.

$$K = \frac{\text{power into network}}{\text{power out of network}}$$

$$K_{min} = \frac{2Z_1}{Z_2} - 1 + 2\sqrt{\frac{Z_1}{Z_2} \left( \frac{Z_1}{Z_2} - 1 \right)} \quad (17.1)$$

where  $K_{min}$  = minimum possible value of  $K$  for particular ratio of  $Z_1$  to  $Z_2$ . The minimum possible values of the power ratio  $K$  as a function of  $Z_1/Z_2$  are given in Fig. 17.3. There is no maximum value for the power ratio. It should be noted that  $Z_1$  is always taken as the larger impedance and can be either the input or output impedance.

Since these networks can be made to have unequal input and output impedances, they are frequently used for impedance matching even though there is an associated power loss.

For the balanced and unbalanced T networks shown in Fig. 17.1 where  $Z_1 \geq Z_2$ ,

$$R_1 = \frac{Z_1(K+1) - 2\sqrt{KZ_1Z_2}}{K-1} \quad (17.2)$$

$$R_2 = \frac{Z_2(K+1) - 2\sqrt{KZ_1Z_2}}{K-1} \quad (17.3)$$

$$R_3 = \frac{2\sqrt{KZ_1Z_2}}{K-1} \quad (17.4)$$

(T)

and if  $Z_1 = Z_2$ ,

$$R_1 = R_2 = Z_1 \left( \frac{\sqrt{K} - 1}{\sqrt{K} + 1} \right) \quad (17.5)$$

$$R_3 = \frac{2Z_1 \sqrt{K}}{K - 1} \quad (17.6)$$

For the balanced and unbalanced  $\pi$  networks shown in Fig. 17.2 where  $Z_1 \geq Z_2$ , the values of  $R_1$ ,  $R_2$ , and  $R_3$  can be determined from Eqs. (17.7) to (17.11).

$$R_1 = \frac{(K - 1)Z_1 \sqrt{Z_2}}{(K + 1) \sqrt{Z_2} - 2 \sqrt{KZ_1}} \quad (17.7)$$

$$R_2 = \frac{(K - 1)Z_2 \sqrt{Z_1}}{(K + 1) \sqrt{Z_1} - 2 \sqrt{KZ_2}} \quad (17.8)$$

$$R_3 = \frac{K - 1}{2} \sqrt{\frac{Z_1 Z_2}{K}} \quad (17.9)$$

and if  $Z_1 = Z_2$ ,

$$R_1 = R_2 = Z_1 \left( \frac{\sqrt{K} + 1}{\sqrt{K} - 1} \right) \quad (17.10)$$

$$R_3 = \frac{Z_1(K - 1)}{2 \sqrt{K}} \quad (17.11)$$

**Example 17.1**

Design a network to match a 500-ohm generator to a 200-ohm load with the minimum possible power loss.

**Solution**

1. Determine the ratio of  $Z_1$  to  $Z_2$  and the minimum possible value of the ratio of network input power to network output power.

$$\frac{Z_1}{Z_2} = \frac{500}{200} = 2.50$$

From Eq. (17.1)

$$K_{min} = 2 \times 2.50 - 1 + 2 \sqrt{2.50(2.50 - 1)} = 7.87$$

2. Determine the type of network to be used and the network values.

Since the type of network was not specified, an arbitrary choice might be an unbalanced T. It was stated that the network loss should be a minimum, therefore  $K$  must equal 7.87. In Fig. 17.3 it is stated that  $R_2 = 0$  ohms when  $K$  is equal to its minimum value; therefore, it is only necessary to determine  $R_1$  and  $R_3$ .

From Eqs. (17.2) and (17.3)

$$R_1 = \frac{500(7.87 + 1) - 2 \sqrt{7.87 \times 500 \times 200}}{7.87 - 1} = 387 \text{ ohms}$$

$$R_3 = \frac{2 \sqrt{7.87 \times 500 \times 200}}{7.87 - 1} = 258 \text{ ohms}$$

(Refer to Fig. 17.4.)

3. Determine the loss in the network.

$$\text{Loss} = 10 \log_{10} K = 8.96 \text{ db}$$

introduce a fixed amount of attenuation. They can be designed to have

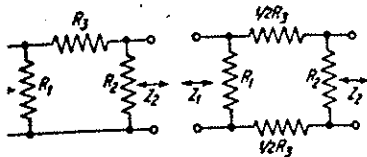


Fig. 17.2.  $\pi$  and O attenuator networks.

and to provide different amounts of

are shown in Figs. 17.1 and 17.2. For impedances there is an associated minimum value of the ratio of input power to the attenuator to output power from the attenuator which can be realized.

$$K = \frac{\text{power into network}}{\text{power out of network}}$$

$$K_{min} = \frac{2Z_1}{Z_2} - 1 + 2 \sqrt{\frac{Z_1}{Z_2} \left( \frac{Z_1}{Z_2} - 1 \right)} \quad (17.1)$$

where  $K_{min}$  = minimum possible value of  $K$  for particular ratio of  $Z_1$  to  $Z_2$ . The minimum possible values of the power ratio  $K$  as a function of  $Z_1/Z_2$  are given in Fig. 17.3. There is no maximum value for the power ratio. It should be noted that  $Z_1$  is always taken as the larger impedance and can be either the input or output impedance.

Since these networks can be made to have unequal input and output impedances, they are frequently used for impedance matching even though there is an associated power loss.

For the balanced and unbalanced T networks shown in Fig. 17.1 where  $Z_1 \geq Z_2$ , determined from Eqs. (17.2) to (17.6).

$$R_1 = \frac{2 \sqrt{KZ_1Z_2}}{K - 1} \quad (17.2)$$

$$R_2 = \frac{2 \sqrt{KZ_1Z_2}}{K - 1} \quad (17.3)$$

$$R_3 = \frac{2 \sqrt{KZ_1Z_2}}{K - 1} \quad (17.4)$$

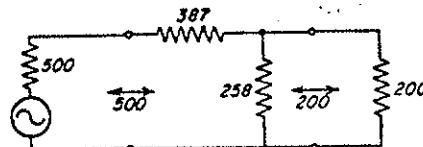


Fig. 17.4. Circuit for Example 17.1.

**Example 17.2**

Design an unbalanced  $\pi$  attenuator with a loss of 20 db ( $K = 100$ ) to operate between a 200-ohm line and a 500-ohm line.

**Solution**

1. Determine the ratio of  $Z_1$  to  $Z_2$  and the minimum possible value of  $K$ .

$$\frac{Z_1}{Z_2} = \frac{500}{200} = 2.50$$

From Example 17.1,  $K$  must be equal to or greater than 7.87.

2. Determine the network values.

Network was specified as an unbalanced  $\pi$  (see Fig. 17.2), and  $K$  is equal to 100. From Eqs. (17.7) to (17.9)

(17.7)

$$R_1 = \frac{(100 - 1)500 \sqrt{200}}{(100 + 1) \sqrt{200} - 2 \sqrt{100 \times 500}} = 714 \text{ ohms}$$

$$R_2 = \frac{(100 - 1)200 \sqrt{500}}{(100 + 1) \sqrt{500} - 2 \sqrt{100 \times 200}} = 224 \text{ ohms}$$

$$R_3 = \frac{(100 - 1)}{2} \sqrt{\frac{200 \times 500}{100}} = 1,567 \text{ ohms}$$

(See Fig. 17.5.)

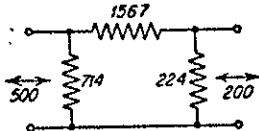


Fig. 17.5. Circuit for Example 17.2.

**17.2. Amplitude Equalizers.** Amplitude equalizers have an insertion loss which varies as some desired function of frequency, and consequently they are employed in electronic circuitry as a means of establishing or correcting the circuit gain characteristics.

In "Motion Picture Sound Engineering," Kimball<sup>1</sup> has provided an excellent treatment of amplitude equalizers, and the material in this section is based on his work.

On the assumption that an amplitude equalizer operates from a source impedance  $R_s$  and into a load impedance  $R_o$ , it is possible to design several equalizers having different configurations but which provide exactly the same attenuation characteristics as a function of frequency. The seven specific configurations for which design information is presented are:

1. Series-impedance type
2. Shunt-impedance type
3. Full-series type
4. Full-shunt type
5. T type
6. Bridged-T type
7. Lattice type

Shown in Fig. 17.6 are the required variations in these seven basic configurations for obtaining the type of attenuation characteristics indicated by the insertion loss curves in each column. The configurations in the last three rows have constant input and output impedances as a function of frequency, and the types in rows 3 and 4 have a constant input impedance.

Although two examples are given to aid in the use of the design data, the following suggestions are included to further help in the design of the different types of equalizers.

<sup>1</sup> Harry Kimball, "Motion Picture Sound Engineering," Chap. 16. D. Van Nostrand Company, Inc., Princeton, N.J., 1938.

COLUMN	I
$Z_1$ $Z_2$	
ROWS	
SERIES IMPEDANCE	
SHUNT IMPEDANCE	
FULL SERIES	
FULL SHUNT	
BRIDGED T	
T TYPE	
LATTICE TYPE	

FIG. 17.6. Network configurations for amplitude equalizers. From "Motion Picture Sound Engineering," Harry Kimball, D. Van Nostrand Company, Inc., Princeton, N.J., 1938.

anced  $\pi$  attenuator with a loss of 20 db ( $K = 100$ ) to operate b  
500-ohm line.  
ratio of  $Z_1$  to  $Z_2$  and the minimum possible value of  $K$ .  
 $\frac{Z_1}{Z_2} = \frac{500}{200} = 2.50$   
must be equal to or greater than 7.87.  
work values.  
d as an unbalanced  $\pi$  (see Fig. 17.2), and  $K$  is equal to 100.  
17.9)

$$R_1 = \frac{(100 - 1)500 \sqrt{200}}{(100 + 1) \sqrt{200} - 2 \sqrt{100 \times 500}} = 714 \text{ ohms}$$

$$R_2 = \frac{(100 - 1)200 \sqrt{500}}{(100 + 1) \sqrt{500} - 2 \sqrt{100 \times 200}} = 224 \text{ ohms}$$

$$R_3 = \frac{(100 - 1)}{2} \sqrt{\frac{200 \times 500}{100}} = 1,567 \text{ ohms}$$

(See Fig. 17.

5. Amplitude equalizers have an insertion loss whi  
on of frequency, and consequently they are employ  
means of establishing or correcting the circuit ga  
Engineering," Kimball<sup>1</sup> has provided an excell  
ers, and the material in this section is based on h

plitude equalizer operates from a source impedanc  
it is possible to design several equalizers having  
h provide exactly the same attenuation character  
The seven specific configurations for which design

variations in these seven basic configurations  
characteristics indicated by the insertion loss  
ations in the last three rows have constant  
tion of frequency, and the types in rows 3  
in the use of the design data, the following  
he design of the different types of equalizers.  
Engineering," Chap. 16. D. Van Nostrand

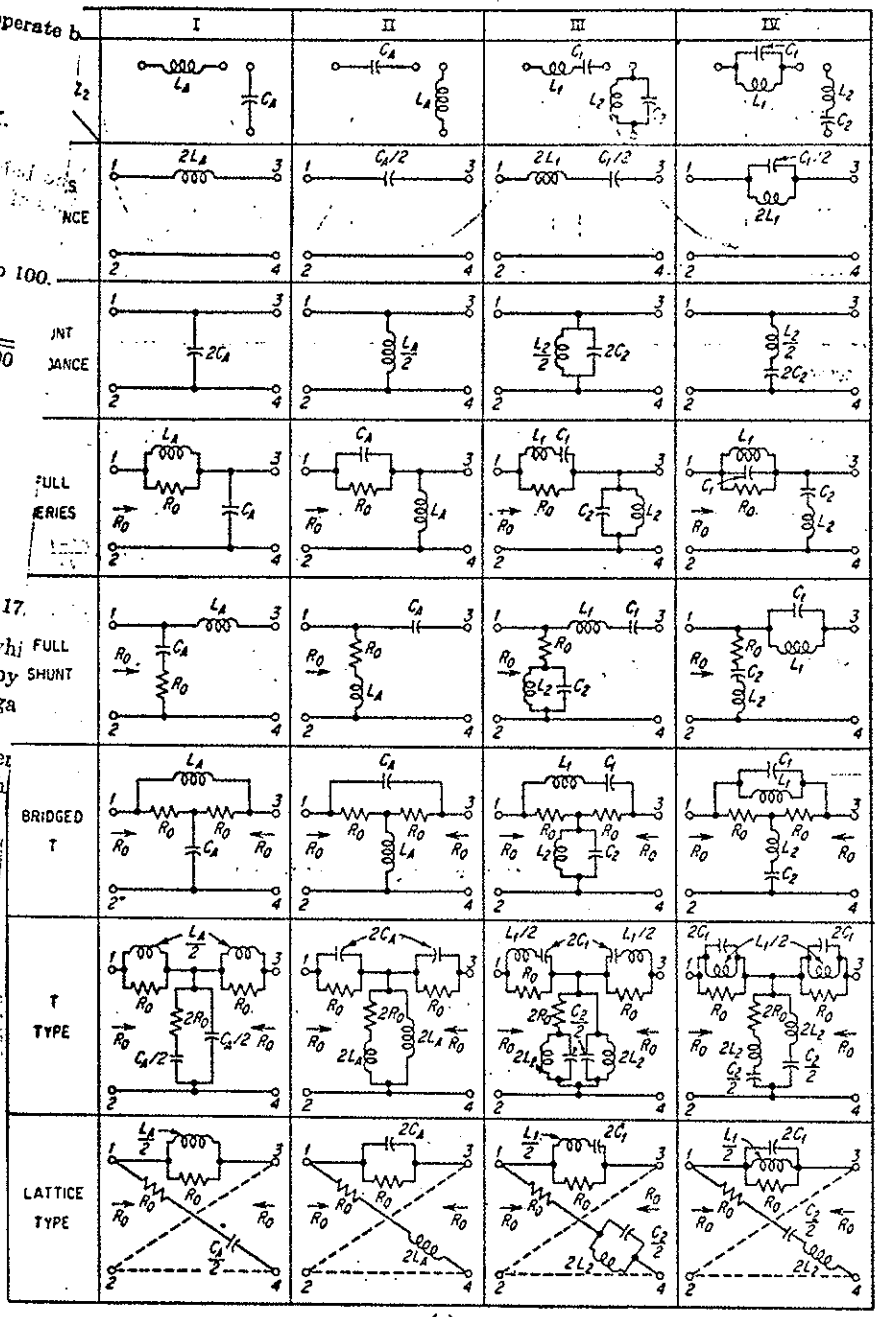
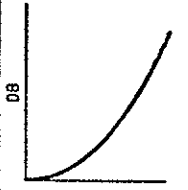
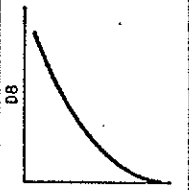
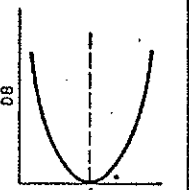
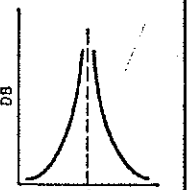
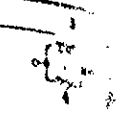


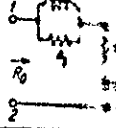

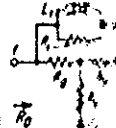
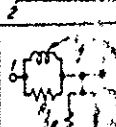
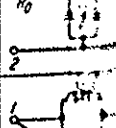
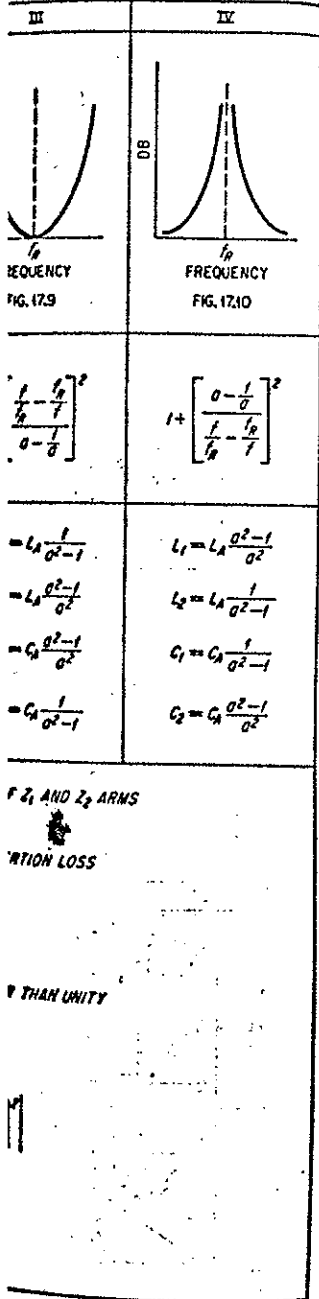


Fig. 17.6. Network configurations and formulas for attenuation equalizers. (From "Motion Picture Sound Engineering," by Research Council of the Academy of Motion Picture Arts and Sciences, copyright 1938, D. Van Nostrand Company, Inc.)

COLUMN	I	II	III	IV
INSERTION LOSS CHARACTERISTIC				
REFER TO	FIG. 17.7	FIG. 17.8	FIG. 17.9	FIG. 17.10
CURRENT RATIO $\left[\frac{I_1}{I_2}\right]^2$	$1 + \left[\frac{f}{f_0}\right]^2$	$1 + \left[\frac{f_0}{f}\right]^2$	$1 + \left[\frac{\frac{f}{f_r} - \frac{f_r}{f}}{a - \frac{1}{a}}\right]^2$	$1 + \left[\frac{a - \frac{1}{a}}{\frac{f}{f_r} - \frac{f_r}{f}}\right]^2$
DESIGN FORMULAE	$L_A = \frac{R_0}{2\pi f_0} = \frac{R_0}{\omega_0}$ $C_A = \frac{1}{2\pi f_0 R_0} = \frac{1}{\omega_0 R_0}$ $f_0 = \frac{1}{2\pi \sqrt{L_A C_A}}$ $R_0 = \sqrt{\frac{L_A}{C_A}}$		$L_1 = L_A \frac{1}{a^2 - 1}$ $L_2 = L_A \frac{a^2 - 1}{a^2}$ $C_1 = C_A \frac{a^2 - 1}{a^2}$ $C_2 = C_A \frac{1}{a^2 - 1}$	$L_1 = L_A \frac{a^2 - 1}{a^2}$ $L_2 = L_A \frac{1}{a^2 - 1}$ $C_1 = C_A \frac{1}{a^2 - 1}$ $C_2 = C_A \frac{a^2 - 1}{a^2}$
NOTES	<p><math>f_r</math> = RESONANT FREQUENCY OF <math>Z_1</math> AND <math>Z_2</math> ARMS</p> <p><math>f_0</math> = FREQUENCY OF 3 DB INSERTION LOSS</p> <p><math>f</math> = ANY FREQUENCY</p> <p><math>a = \frac{f_r}{f_0}</math> = DEFINED AS GREATER THAN UNITY</p> <p><math>R_0</math> = EQUALIZER RESISTANCE</p> <p>INSERTION LOSS = <math>10 \log \left[\frac{I_1}{I_2}\right]^2</math></p> <p><math>L</math> = INDUCTANCE IN HENRIES</p> <p><math>C</math> = CAPACITANCE IN FARADS</p>			

(b)  
Fig. 17.6. (Continued)

COLUMN	
ROWS	
SERIES IMPEDANCE	
SHUNT IMPEDANCE	
FULL SERIES	
FULL SHUNT	
BRIDGED T	
T TYPE	
LATTICE TYPE	



COLUMN	V	VI	VII	VIII
Z <sub>1</sub> AND Z <sub>2</sub>				
ROWS				
SERIES IMPEDANCE				
SHUNT IMPEDANCE				
FULL SERIES				
FULL SHUNT				
BRIDGED T				
T TYPE				
LATTICE TYPE				

(c)  
FIG. 17.6. (Continued)

COLUMN	I	II	III	IV
INSERTION LOSS CHARACTERISTIC				
REFER TO	FREQUENCY FIG. 17.11	FREQUENCY FIG. 17.12	FREQUENCY FIG. 17.13	FREQUENCY FIG. 17.14
CURRENT RATIO	$1 + \frac{K^2 - 1}{1 + K \left(\frac{f}{f_b}\right)^2}$	$1 + \frac{K^2 - 1}{1 + K \left(\frac{f}{f_b}\right)^2}$	$1 + \frac{K^2 - 1}{1 + K \left[\frac{f/f_b}{b - f/f_b}\right]^2}$	$1 + \frac{K^2 - 1}{1 + K \left[\frac{f/f_b}{b - f/f_b}\right]^2}$
DESIGN FORMULAE	$L_1 = L_0 \frac{K-1}{\sqrt{K}}$ $L_2 = L_0 \frac{\sqrt{K}}{K-1}$ $C_1 = C_0 \frac{\sqrt{K}}{K-1}$ $C_2 = C_0 \frac{K-1}{\sqrt{K}}$	$L_1 = L_0 \frac{K-1}{\sqrt{K}} \frac{b^2-1}{b^2}$ $L_2 = L_0 \frac{\sqrt{K}}{K-1} \frac{1}{b^2-1}$ $C_1 = C_0 \frac{\sqrt{K}}{K-1} \frac{1}{b^2-1}$ $C_2 = C_0 \frac{K-1}{\sqrt{K}} \frac{b^2-1}{b^2}$	$L_1 = L_0 \frac{K-1}{\sqrt{K}} \frac{1}{b^2-1}$ $L_2 = L_0 \frac{\sqrt{K}}{K-1} \frac{b^2-1}{b^2}$ $C_1 = C_0 \frac{\sqrt{K}}{K-1} \frac{b^2-1}{b^2}$ $C_2 = C_0 \frac{K-1}{\sqrt{K}} \frac{1}{b^2-1}$	
FOR ALL NETWORKS				
$R_0 = \sqrt{\frac{L_0}{C_0}}$ $R_1 = R_0(K-1)$ $R_2 = R_0 \frac{1}{K-1}$ $R_3 = R_0 \frac{K}{K-1}$ $R_3 = R_0 \frac{K-1}{K+1}$ $R_4 = R_0 \frac{K+1}{K-1}$ $R_5 = R_0 \frac{K-1}{K}$ $L_0 = \frac{R_0}{2\pi f_b} = \frac{R_0}{\omega_b}$ $C_0 = \frac{1}{2\pi f_b R_0} = \frac{1}{\omega_b R_0}$ $f_b = \frac{1}{2\pi \sqrt{L_0 C_0}}$				
$f_R$ = RESONANT FREQUENCY OF $Z_1$ AND $Z_2$ ARMS $f$ = ANY FREQUENCY INSERTION LOSS = $10 \log \left[ \frac{I_1}{I_2} \right]^2$ PAD LOSS = MAXIMUM LOSS = $20 \log K$ $b = \frac{f_R}{f_b}$ = DEFINED AS GREATER THAN UNITY $L$ = INDUCTANCE IN HENRIES $f_b$ = FREQUENCY OF ONE-HALF PAD LOSS $C$ = CAPACITANCE IN FARADS $R_0$ = EQUALIZER RESISTANCE				

(d)  
FIG. 17.6. (Continued)

For the equalizer loss at some frequency is applicable, the

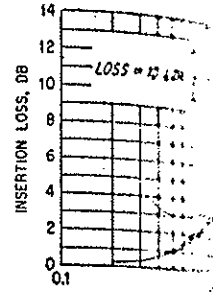


FIG. 17.7. Attenuation works shown in section (From "Motion Picture Arts and Sciences" by Research Council of Picture Arts and Sciences, Van Nostrand Company)

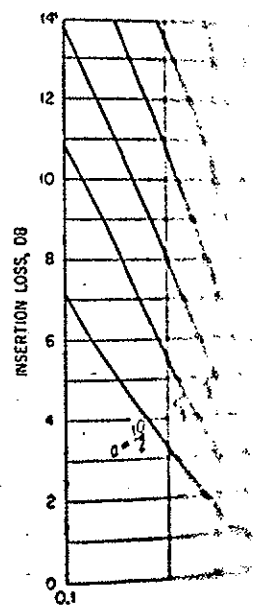


FIG. 17.9. Attenuation works shown in section (From "Motion Picture Arts and Sciences" by Research Council of Picture Arts and Sciences, Van Nostrand Company)

The value of  $f_0$ , which can then be determined. When working with infinite insertion loss value of  $f/f_0$  must be approxi-

INDEX

$\frac{K^2-1}{\left[\frac{f}{f_0} - \frac{f_0}{f}\right]^2}$	$1 + \frac{K^2-1}{\left[\frac{b-\frac{1}{b}}{f} - \frac{f}{f_r}\right]^2}$
$L_1 = L_0 \frac{K-1}{\sqrt{K}} \frac{1}{b^2-1}$	
$L_2 = L_0 \frac{\sqrt{K}}{K-1} \frac{b^2-1}{b^2}$	
$C_1 = C_0 \frac{\sqrt{K}}{K-1} \frac{b^2-1}{b^2}$	
$C_2 = C_0 \frac{K-1}{\sqrt{K}} \frac{1}{b^2-1}$	
$R_0 = R_0 \frac{K}{K-1}$	
$R_0 = \frac{1}{f_0 L_0 C_0}$	
$f = \text{ANY FREQUENCY}$	
$\text{MAX LOSS} = \text{MAXIMUM LOSS} = 20 \text{ LOG } K$	
$L = \text{INDUCTANCE IN HENRIES}$	
$C = \text{CAPACITANCE IN FARADS}$	
$R = \text{EQUALIZER RESISTANCE}$	

For the equalizers treated in columns I and II of Fig. 17.6, the desired insertion loss at some frequency  $f$  must be specified. From either Fig. 17.7 or 17.8, whichever is applicable, this insertion loss can then be associated with a specific value of  $f/f_a$ .

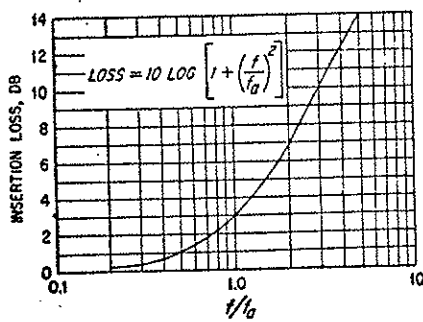


FIG. 17.7. Attenuation characteristics of networks shown in column I of Fig. 17.6. (From "Motion Picture Sound Engineering," by Research Council of the Academy of Motion Picture Arts and Sciences, copyright 1938, D. Van Nostrand Company, Inc.)

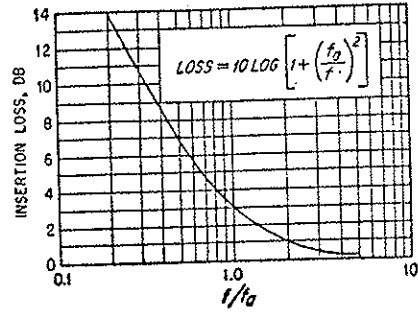


FIG. 17.8. Attenuation characteristics of networks shown in column II of Fig. 17.6. (From "Motion Picture Sound Engineering," by Research Council of the Academy of Motion Picture Arts and Sciences, copyright 1938, D. Van Nostrand Company, Inc.)

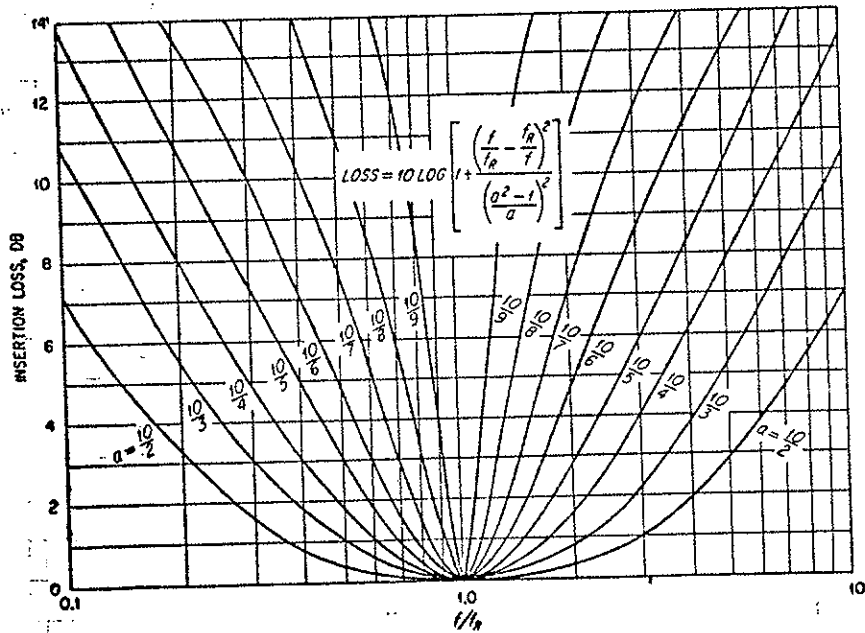


FIG. 17.9. Attenuation characteristics of networks shown in column III of Fig. 17.6. (From "Motion Picture Sound Engineering," by Research Council of the Academy of Motion Picture Arts and Sciences, copyright 1938, D. Van Nostrand Company, Inc.)

The value of  $f_a$ , which is required for the calculation of the equalizer circuit values, can then be determined since the values of  $f$  and  $f/f_a$  are known.

When working with equalizers of the types shown in columns III and IV of Fig. 17.6, the frequency of resonance  $f_r$  within the equalizer (associated with zero and infinite insertion losses, respectively) and the desired insertion loss at some specific value of  $f/f_r$  must be specified so that the proper attenuation curve and the asso-

$a = 1.5573$   
 $\rightarrow 6.45 \text{ dB/OCTAVE}$



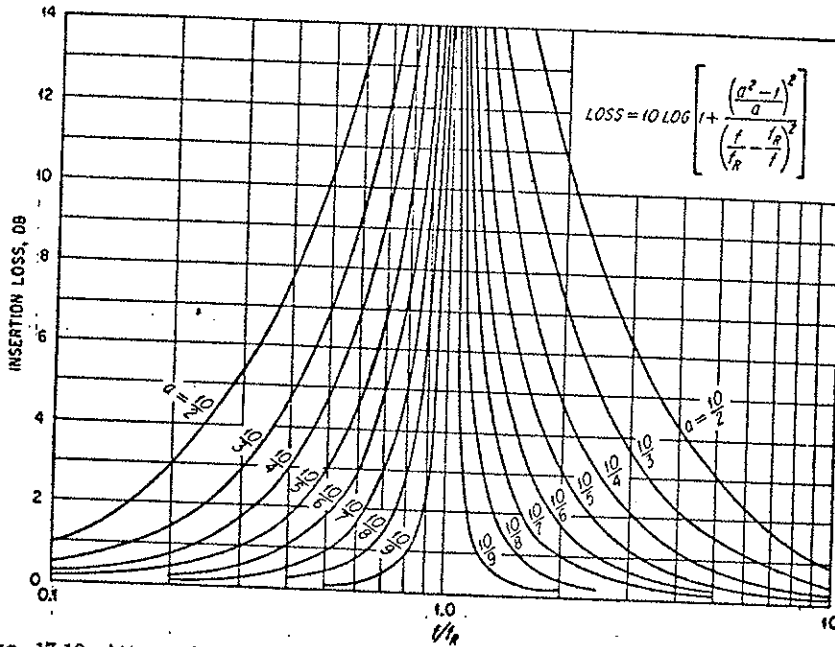


FIG. 17.10. Attenuation characteristics of networks shown in column IV of Fig. 17.6. (From "Motion Picture Sound Engineering," by Research Council of the Academy of Motion Picture Arts and Sciences, copyright 1938, D. Van Nostrand Company, Inc.)

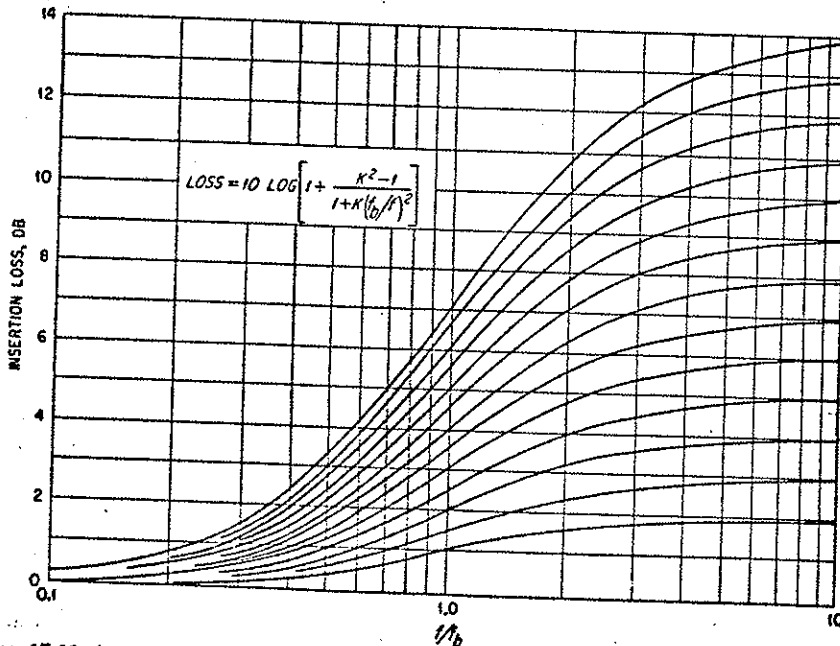


FIG. 17.11. Attenuation characteristics of networks shown in column V of Fig. 17.6. (From "Motion Picture Sound Engineering," by Research Council of the Academy of Motion Picture Arts and Sciences, copyright 1938, D. Van Nostrand Company, Inc.)

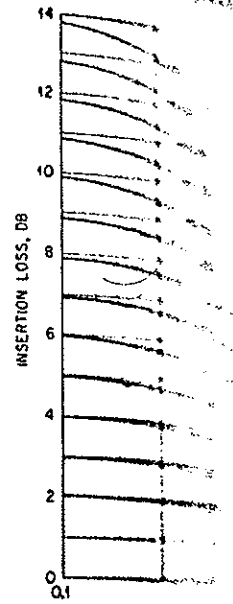


FIG. 17.12. Attenuation characteristics of networks shown in column IV of Fig. 17.6. (From "Motion Picture Sound Engineering," by Research Council of the Academy of Motion Picture Arts and Sciences, copyright 1938, D. Van Nostrand Company, Inc.)

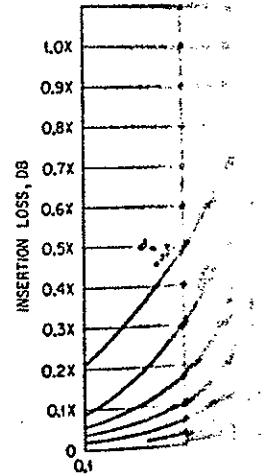


FIG. 17.13. Attenuation characteristics of networks shown in column V of Fig. 17.6. (From "Motion Picture Sound Engineering," by Research Council of the Academy of Motion Picture Arts and Sciences, copyright 1938, D. Van Nostrand Company, Inc.)

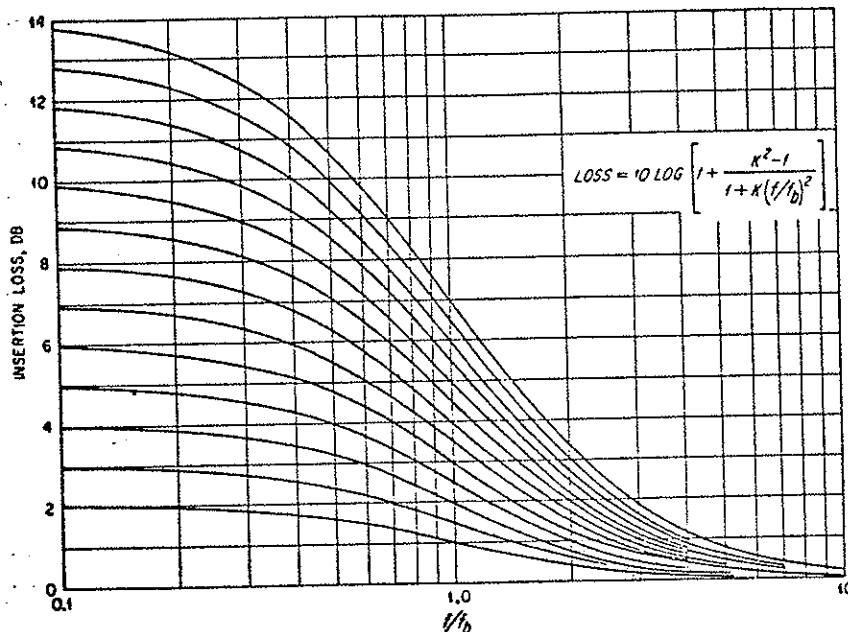
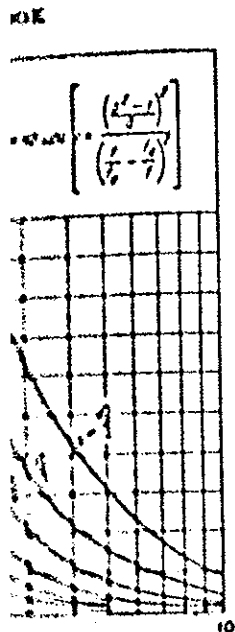


FIG. 17.12. Attenuation characteristics of networks shown in column VI of Fig. 17.6 (Figs. 17.12, 17.13, and 17.14 from "Motion Picture Sound Engineering," by Research Council of the Academy of Motion Picture Arts and Sciences, copyright 1938, D. Van Nostrand Company, Inc.)

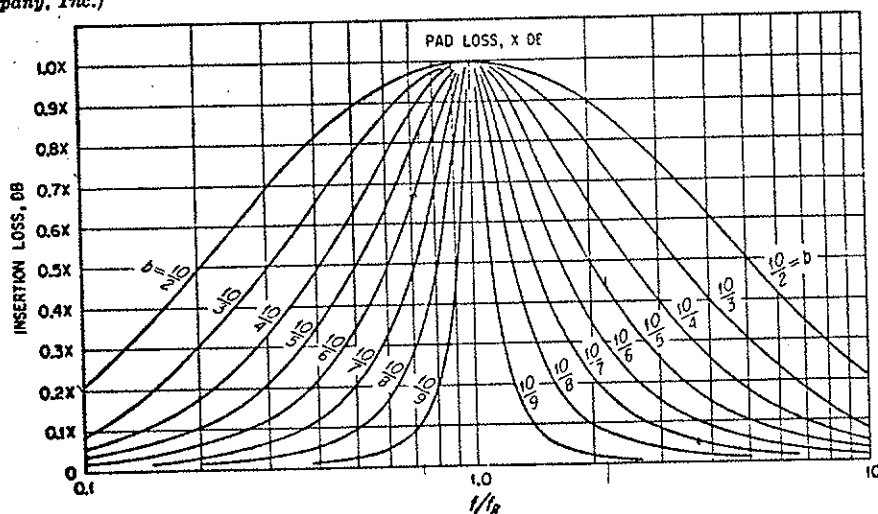
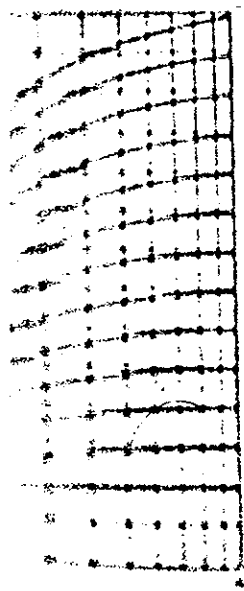


FIG. 17.13. Attenuation characteristics of networks shown in column VII of Fig. 17.6.

ciated value of  $a$  can be determined from either Fig. 17.9 or 17.10, whichever is applicable. The value of  $f_s$ , which is also required for the calculation of the equalizer circuit values, can be determined by dividing  $f_R$  by  $a$ .

The first step in the design of equalizers in columns V and VI of Fig. 17.6 is to specify the maximum desired loss and the loss at some specific frequency  $f$ . The loss at  $f$  can then be associated with a specific value of  $f/f_b$  on the curve having the desired maximum loss in either Fig. 17.11 or 17.12, whichever is applicable. The value of  $f_b$

Figure 17.13 of Fig. 17.6. The curves are for values of a ranging from 1/2 to 10.

which is required for the calculation of the equalizer circuit values, can then be determined since the values of  $f$  and  $f/f_0$  are known.

To design equalizers shown in columns VII and VIII of Fig. 17.6, the maximum insertion loss, the frequency of resonance  $f_R$  within the equalizer (associated with the

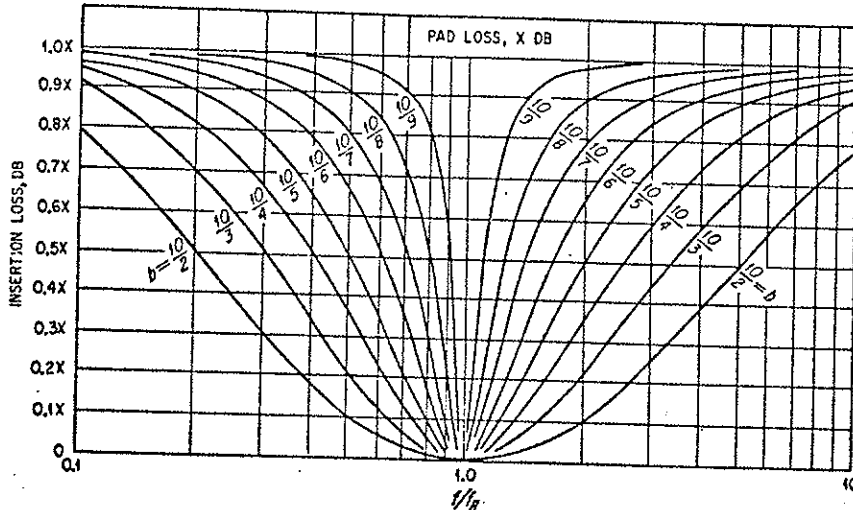


Fig. 17.14. Attenuation characteristics of networks shown in column VIII of Fig. 17.6.

maximum and zero insertion losses, respectively), and the desired insertion loss at some value of  $f/f_R$  must be specified. It is then possible to establish the proper curve and the associated value of  $b$  from either Fig. 17.13 or 17.14, whichever is applicable.

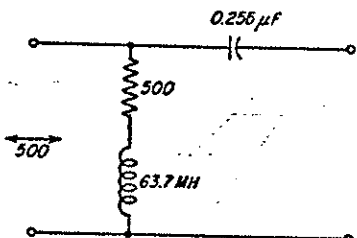


Fig. 17.15. Full shunt equalizer for Example 17.3.

loss of 4 db at 1 kc and an input impedance of 500 ohms.

**Solution**

From Fig. 17.8  $f/f_0 = 0.8$  for an insertion loss of 4 db. Therefore,

$$f_0 = \frac{f}{0.8} = \frac{1,000}{0.8} = 1,250 \text{ cycles}$$

$$R_0 = 500 \text{ ohms (from statement of problem)}$$

$$L_0 = \frac{500}{2 \times 3.14 \times 1,250} = 63.7 \times 10^{-3} \text{ henry, or } 63.7 \text{ mh}$$

$$C_0 = \frac{1}{2 \times 3.14 \times 1,250 \times 500} = 0.255 \times 10^{-6} \text{ farad, or } 0.255 \mu F$$

(See Figs. 17.15 and 17.16.)

Fig. 17.16. Insertion loss characteristics.

**Example 17.4**

Design an amplitude equalizer of the type shown in column VIII of Fig. 17.6 and of the type shown in column VII of Fig. 17.6 (frequency of equalizer resonance is 3,500 cps and the input impedance of 200 ohms).

**Solution**

1. Determine  $f_R$ ,  $b$ ,  $f_0$  and  $k$ .

$$f_R = 3,500 \text{ cps}$$

As being equal to 20 db at 3,500 cps and the input impedance of 200 ohms. The values of  $b$  and  $k$  are determined as follows:

From Fig. 17.14, for a loss of 20 db,  $b = 10/2 = 5$ .

From Fig. 17.14, for a loss of 20 db,  $k = 10/2 = 5$ .

2. Determine the values of  $L_0$  and  $C_0$ . Refer to Fig. 17.4.

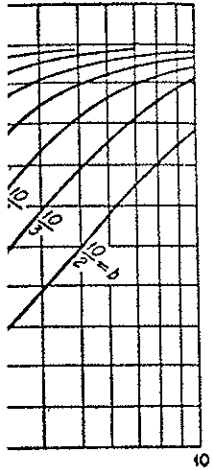
$$R_0 = 200 \text{ ohms}$$

$$L_0 = \frac{200}{2 \times 3.14 \times 3,500} = 2.27 \times 10^{-3} \text{ henry, or } 2.27 \text{ mh}$$

$$C_0 = \frac{1}{2 \times 3.14 \times 3,500 \times 200} = 2.27 \times 10^{-8} \text{ farad, or } 2.27 \mu F$$

, can then be deter-

17.6, the maximum (associated with the



an VIII of Fig. 17.6.

red insertion loss at  
lish the proper curve  
ichever is applicable.  
also required for the  
ircuit values can be  
by  $b$ .

ns V to VIII, the fre-  
at which the pad loss  
e maximum loss in  
um loss is 8 db,  $f_b$  is  
: loss is 4 db.

izer of the type shown  
which has an insertion



re,

mh  
or 0.255  $\mu$ f

Fig. 17.15 and 17.16.)

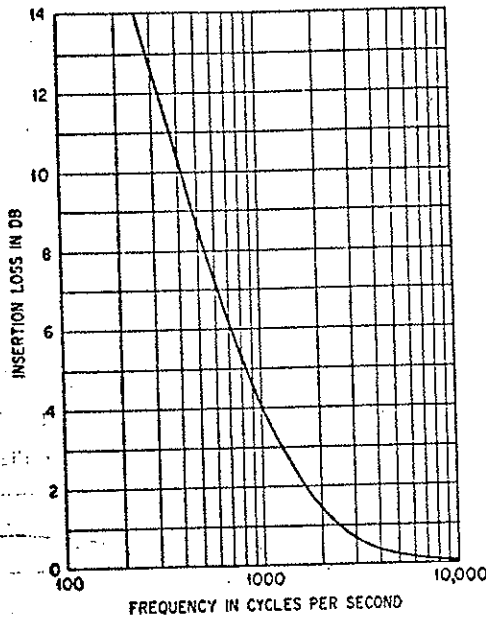


Fig. 17.16. Insertion-loss characteristics of network shown in Fig. 17.15.

**Example 17.4**

Design an amplitude equalizer with attenuation characteristics as indicated in column VIII of Fig. 17.6 and of the bridged-T type which will introduce zero attenuation at 5 kc (frequency of equalizer resonance), 14-db attenuation at 3.5 kc, and 20-db attenuation at frequencies far above and far below 5 kc. The equalizer should have a characteristic impedance of 200 ohms.

**Solution**

1. Determine  $f_R$ ,  $b$ ,  $f_b$ , and  $K$ .

$f_R = 5,000$  cycles (frequency of resonance and no attenuation)

At 3,500 cps,  $f/f_R = 3,500/5,000 = 0.7$ . The maximum attenuation has been specified as being equal to 20 db; therefore,  $X = 20$  db in Fig. 17.14, and the desired attenuation of 14 db at 3,500 cps is equal to  $0.7X$ . The curve for  $b = 10/8$  satisfies these conditions. The values of  $f_b$  and  $K$  are determined as follows:

$$b = \frac{f_R}{f_b}$$

$$f_b = \frac{5,000}{1.25} = 4,000 \text{ cycles}$$

$$20 \log_{10} K = \text{maximum pad loss} = 20$$

$$K = 10$$

2. Determine the values of the elements in the bridged T.  
Refer to Fig. 17.6.

$R_s = 200$  ohms (specified in statement of problem)

$$L_B = \frac{200}{2 \times 3.14 \times 4,000} = 7.96 \times 10^{-3} \text{ henry, or } 7.96 \text{ mh}$$

$$C_B = \frac{1}{2 \times 3.14 \times 4,000 \times 200} = 0.199 \times 10^{-6} \text{ farad, or } 0.199 \mu\text{f}$$

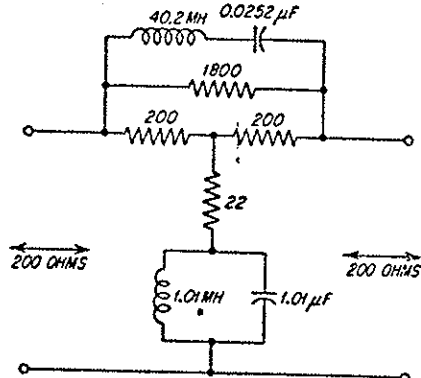


Fig. 17.17. Bridged-T equalizer for Example 17.4.

$$L_1 = 7.96 \times \frac{10 - 1}{\sqrt{10}} \times \frac{1}{1.25^2 - 1} = 40.2 \text{ mh}$$

$$L_2 = 7.96 \times \frac{\sqrt{10}}{10 - 1} \times \frac{1.25^2 - 1}{1.25^2} = 1.01 \text{ mh}$$

$$C_1 = 0.199 \times \frac{\sqrt{10}}{10 - 1} \times \frac{1.25^2 - 1}{1.25^2} = 0.0252 \mu\text{f}$$

$$C_2 = 0.199 \times \frac{10 - 1}{\sqrt{10}} \times \frac{1}{1.25^2 - 1} = 1.01 \mu\text{f}$$

$$R_1 = 200(10 - 1) = 1,800 \text{ ohms}$$

$$R_2 = 200 \times \frac{1}{10 - 1} = 22.2 \text{ ohms}$$

(Refer to Figs. 17.17 and 17.18.)

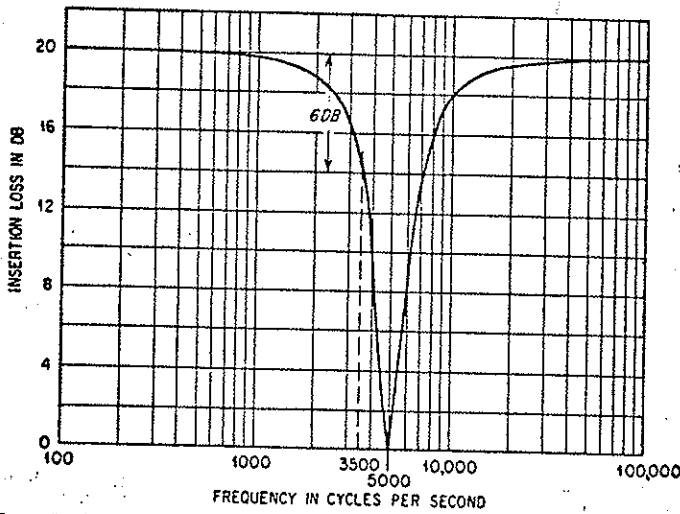


Fig. 17.18. Insertion-loss characteristics of network shown in Fig. 17.17.

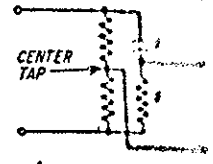
**17.3. Phase Equalizers.** The types of phase equalizers treated are those which theoretically introduce either zero or a fixed amount of attenuation at all frequencies. They can therefore be added to existing circuits for phase correction without distorting the gain characteristics.

The shape of electrical impulses which contain many frequency components can be distorted in passing through an electrical circuit even though the circuit has the same gain for the different frequency components. If such is the case, the distortion is due to unequal transmission delays for the different frequency components. This type of distortion is called *phase distortion* and can be corrected by adding a network which will cause the total transmission period for all frequencies to be identical. The added network must, therefore, be a network in which the phase characteristics can be controlled.

Equal transmission periods for different frequency components through a circuit stipulates that the circuit must introduce either no phase shift or an amount of phase shift which is directly proportional to frequency. This is identical to stating that the transmission period must be either zero or a constant amount at all frequencies.

Four different configurations... It should be noted that the... connections in which the input... connected to each other.

The circuit in Fig. 17.19... constant input and output...



$$f_0 = \frac{1}{2\pi RC}$$

WHERE  $f_0$  IS THE FREQUENCY AT WHICH THE PHASE SHIFT IS A 90-Degree...

Fig. 17.19. Phase equalizer... fixed insertion loss... phase characteristics... the same as for the... shown in Fig. 17.17... output is not loaded.

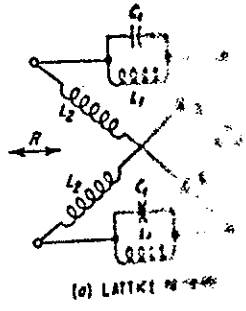


Fig. 17.21. Phase-shift... characteristics.

phase-shift curve for the... impedance.

A center-tapped... resistor in Fig. 17.17... former were acceptable.

The networks... characteristic impedances... attenuation. Figure... be obtained.

The phase equalizer... These networks...

OK

$$\frac{-1}{0} \times \frac{1}{1.25^2 - 1} = 40.2 \text{ mh}$$

$$\frac{0}{1} \times \frac{1.25^2 - 1}{1.25^2} = 1.01 \text{ mh}$$

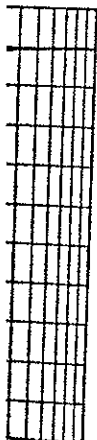
$$\frac{10}{-1} \times \frac{1.25^2 - 1}{1.25^2} = 0.0252 \mu\text{f}$$

$$\frac{-1}{0} \times \frac{1}{1.25^2 - 1} = 1.01 \mu\text{f}$$

$$= 1,800 \text{ ohms}$$

$$= 22.2 \text{ ohms}$$

(Figs. 17.17 and 17.18.)



100,000

In Fig. 17.17.

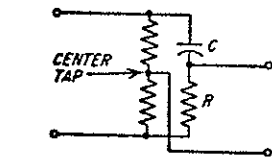
ted are those which on at all frequencies. direction without dis-

r components can be circuit has the same use, the distortion is components. This by adding a network be identical. The hase characteristics

is through a circuit an amount of phase ical to stating that t at all frequencies.

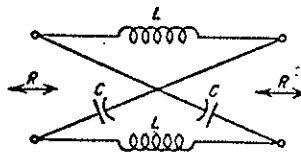
Four different configurations of phase equalizers<sup>1</sup> are shown in Figs. 17.19 to 17.21. It should be noted that the four-terminal networks can be used in only those applications in which the input and output circuits are either both balanced or are in no way connected to each other.

The circuit in Fig. 17.19 introduces an insertion loss of 6 db and does not have constant input and output impedances as a function of frequency. In addition, the



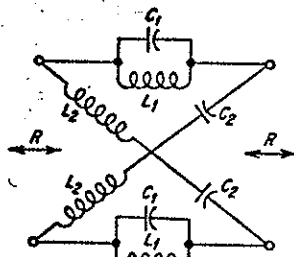
$f_0 = \frac{1}{2\pi RC}$   
WHERE  $f_0$  IS THE FREQUENCY AT WHICH THE PHASE SHIFT  $\beta$  IS EQUAL TO  $-90^\circ$

FIG. 17.19. Phase equalizer with a fixed insertion loss of 6 db. The phase characteristics are exactly the same as for the lattice network shown in Fig. 17.20, provided the output is not loaded.



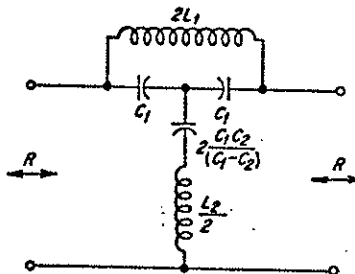
$L = \frac{aR}{2\pi}$       $a = 1/f_0$   
 $C = \frac{L}{R^2}$      WHERE  $f_0$  IS THE FREQUENCY AT WHICH NETWORK PHASE SHIFT IS  $-90^\circ$   
 $\text{TAN } \frac{\beta}{2} = af$

FIG. 17.20. Phase-shift network with zero attenuation. Refer to Fig. 17.22 for phase characteristics.



(a) LATTICE NETWORK

$L_1 = \frac{aR}{2\pi}$       $C_1 = \frac{b}{2\pi aR}$   
 $L_2 = R^2 C_1$       $C_2 = \frac{L_1}{R^2}$   
 $\text{TAN } \frac{\beta}{2} = \frac{af}{1 - b^2 f^2}$



(b) BRIDGED-T EQUIVALENT TO LATTICE IF  $C_1 > C_2$

$b = 1/f_0^2$  WHERE  $f_0$  IS THE FREQUENCY AT WHICH THE NETWORK PHASE SHIFT IS  $-180^\circ$

FIG. 17.21. Phase-shift network with zero attenuation. Refer to Fig. 17.23 for phase characteristics.

phase-shift curve for the circuit, Fig. 17.22, is based on there being no terminating impedance.

A center-tapped transformer secondary winding could be substituted for the resistor in Fig. 17.19, provided the amplitude and phase characteristics of the transformer were acceptable.

The networks shown in Figs. 17.20 and 17.21 have constant input and output characteristic impedances as a function of frequency and provide phase shift without attenuation. Figures 17.22 and 17.23 indicate the phase characteristics which can be obtained.

The phase equalizers shown in Figs. 17.19 to 17.21 introduce a lagging phase shift.

<sup>1</sup> These networks are also referred to as all-pass filters.

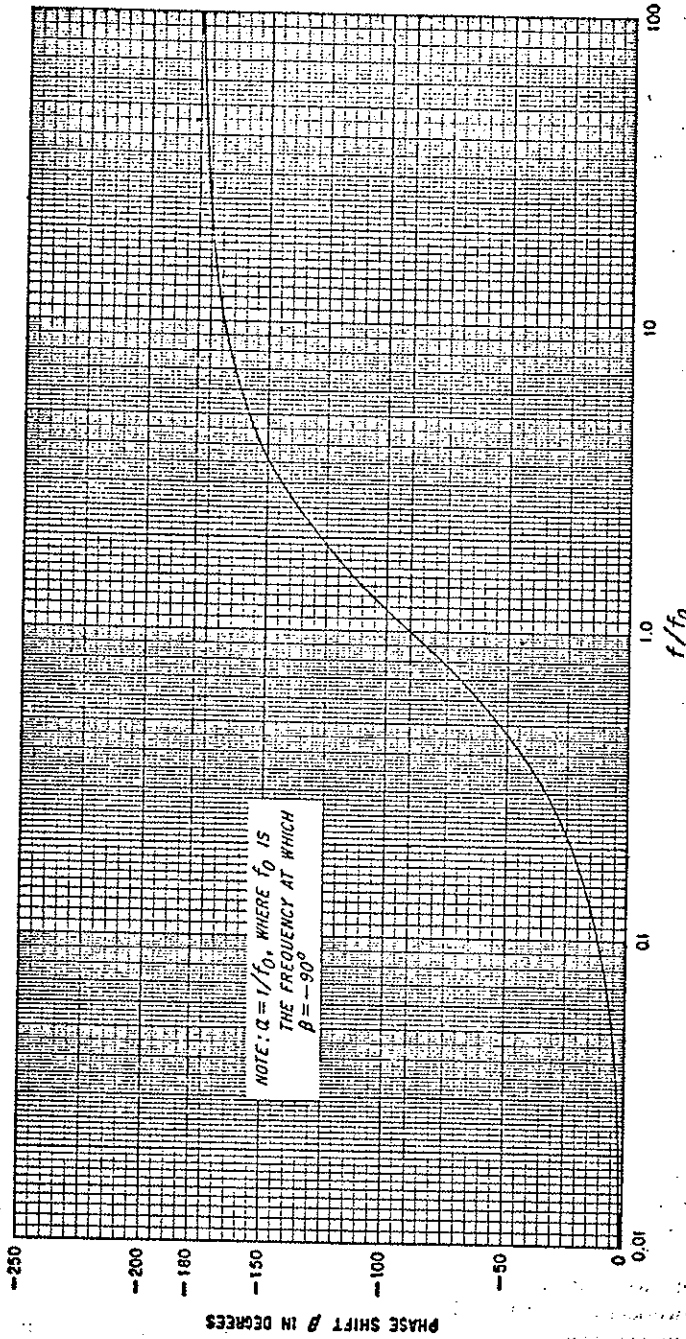


Fig. 17.22. Phase shift in an all-pass filter of the type shown in Fig. 17.20.

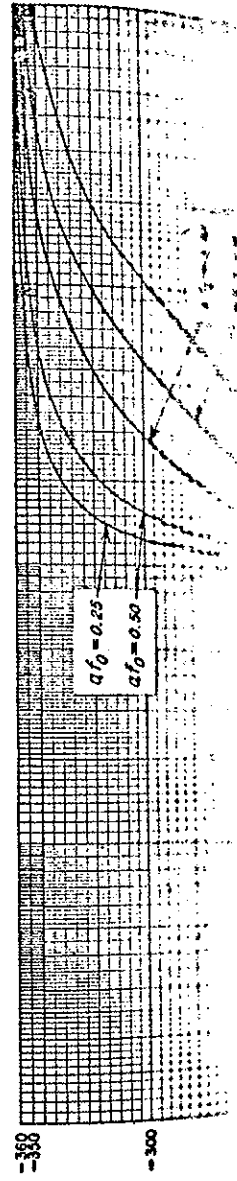


Fig. 17.23. Phase shift in an all-pass filter of the type shown in Fig. 17.20.

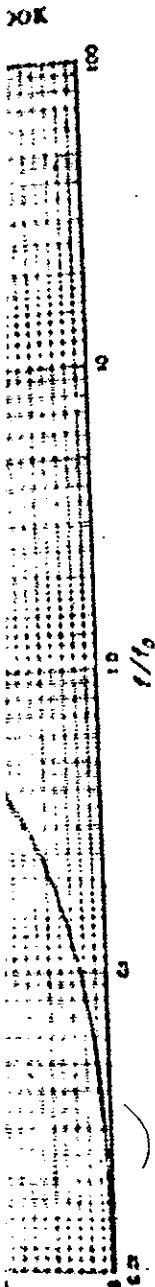


FIG. 17.22. Phase shift in an all-pass filter of the type shown in Fig. 17.20.

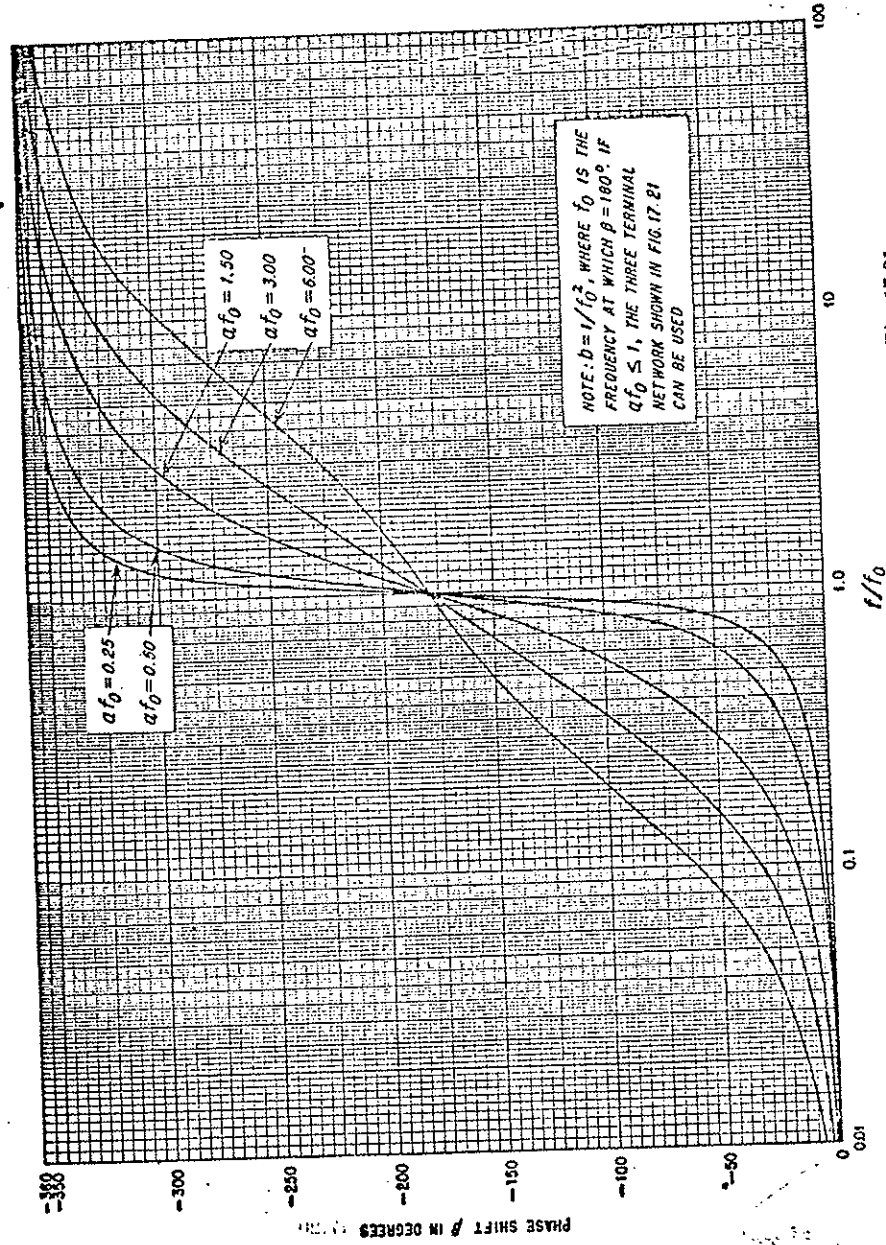


FIG. 17.23. Phase shift in an all-pass filter of the type shown in Fig. 17.21.



**Example 17.5**

Assume that intelligence must be transmitted in the 10- to 20-ke frequency band and that the circuit employed introduces phase shift in accordance with the following tabulation.

Phase Shift	Frequency
-27°	10 kc
-43.5°	15 kc
-63°	20 kc

Design a phase equalizer of the lattice type with a characteristic impedance of 1,000 ohms for use with this circuit.

**Solution**

1. Determine the required phase characteristics of the phase equalizer.

The departure from linear phase shift as a function of frequency for the existing circuit must first be determined. Since the phase shift at 20 kc is -63°, the phase shift at 10 kc should be  $\frac{1}{2} \times -63$ , or -31.5°, and the phase shift at 15 kc should be  $\frac{3}{4} \times -63$ , or -47.25°. The existing network therefore introduces a phase error of +4.5° at 10 kc and +3.75° at 15 kc.

Phase Error	Frequency
+4.5°	10 kc
+3.75°	15 kc
0°	20 kc

The phase equalizer must therefore exhibit the inverse characteristics, i.e.,

Phase Error	Frequency
-4.5°	10 kc
-3.75°	15 kc
0°	20 kc

2. Determine from Figs. 17.22 and 17.23 if the required conditions tabulated in step 1 can be satisfied with either of the networks shown in Figs. 17.20 or 17.21.

Since the network shown in Fig. 17.20 is simpler, the curve shown in Fig. 17.22 should first be examined.

The procedure is to determine if the phase shift in the equalizer at any three values of  $f/f_0$ , which are related in the same proportions as are 10, 15, and 20 kc, will depart from linear phase shift as a function of frequency by the desired amount. A few experimental groups of values of  $f/f_0$  reveal that the phase shifts for  $f/f_0$  equal to 0.4, 0.6, and 0.8 are equal to -43, -61.5, and -77°, respectively, and satisfy the specified requirements. This is true since a phase shift of -77° at  $f/f_0 = 0.8$  requires that the phase shift be -38.5 and -57.75° at  $f/f_0$  equal to 0.4 and 0.6, respectively, for linear phase characteristics. The phase equalizer therefore introduces phase errors of -4.5 and -3.75° when  $f/f_0$  is equal to 0.4 and 0.6, respectively. It should be apparent that the three values of  $f/f_0$ , that is, 0.4, 0.6, and 0.8, correspond to  $f$  being equal to 10, 15, and 20 kc, respectively.

3. Determine  $f_0$  and the values for the lattice elements.

$$\frac{f}{f_0} = 0.8 \text{ (at } f = 20 \text{ kc)}$$

$$f_0 = 25 \text{ kc}$$

From Fig. 17.22

$$\alpha = \frac{1}{25,000} = 4 \times 10^{-5}$$

From Fig. 17.20

$$L = \frac{4 \times 10^{-5} \times 10^3}{2 \times 3.14}$$

$$= 6.37 \times 10^{-3} \text{ henry, or } 6.37 \text{ mh}$$

$$C = \frac{6.37 \times 10^{-3}}{10^6}$$

$$= 6.370 \times 10^{-12} \text{ farad, or } 6,370 \mu\text{f}$$

The lattice network is shown in Fig. 17.24.

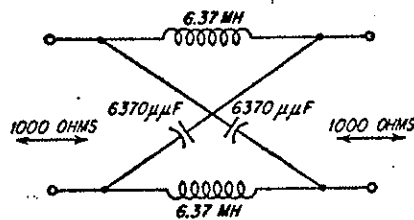


Fig. 17.24. Lattice network for Example 17.5.

# Principles

18.1. Introduction

18.2. System Characteristics

18.3. Transfer Functions

18.4. Methods of Analysis

18.5. Minimum Phase Networks