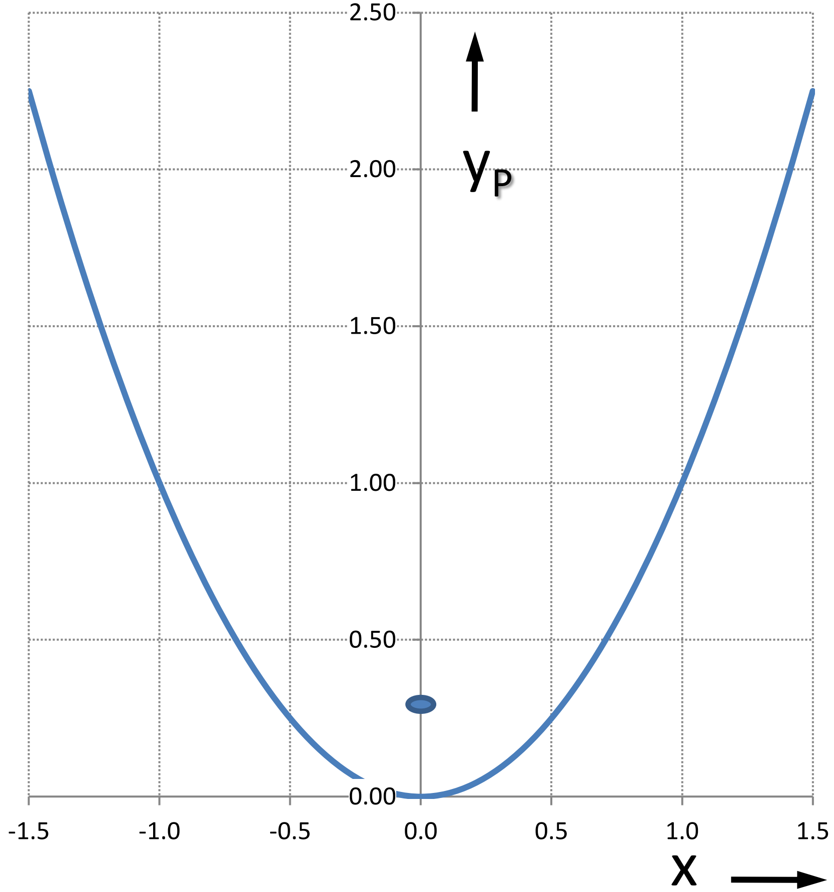
OFFSET-FEED DISH

CALCULATIONS

Parabola

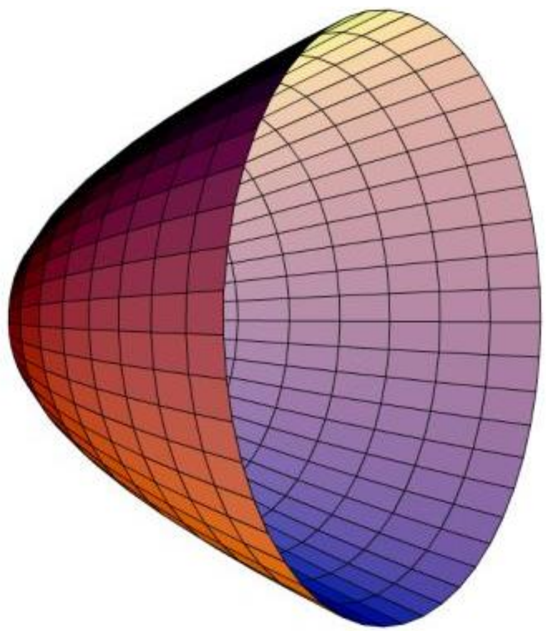


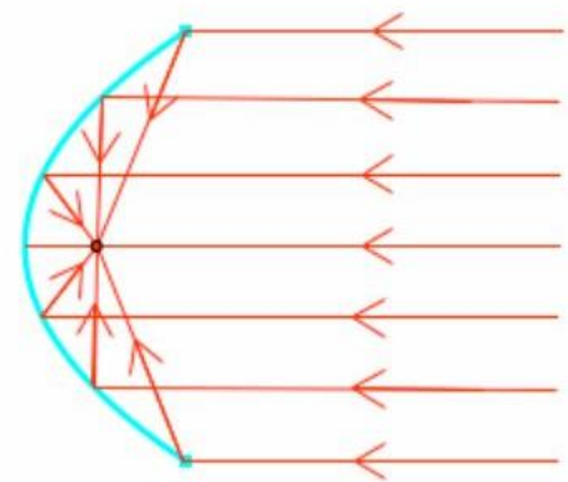
- All parabolas are similar

$$y_P = b\,x^2$$

- Scaling parameter determines:
  - Relative curvature
  - f/D
  - How ‘shallow’ or ‘deep’ the dish

Paraboloid

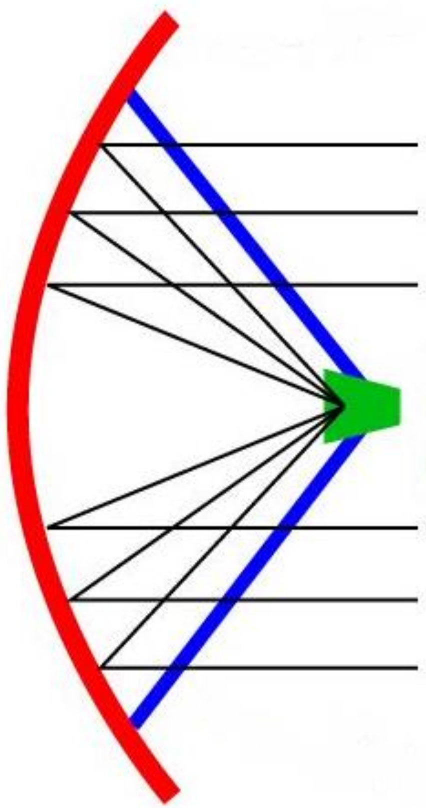




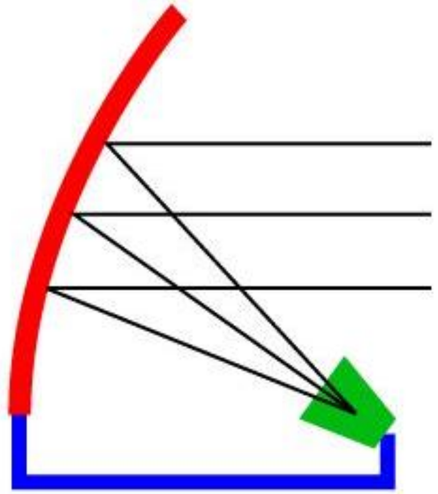
- Plane waves are focused *IN-PHASE* to the focal point
- A paraboloid surface is a parabola rotated about the focal axis
- Dish reflectors are usually defined by the intersection of a parapolid and a plane
  - Axial feed: intersecting plane is perpendicular to focal axis
  - Offset feed: intersecting plane cuts through the origin (end of focal axis)

Single Reflector Dish Feeds

AXIAL FEED



OFFSET FEED

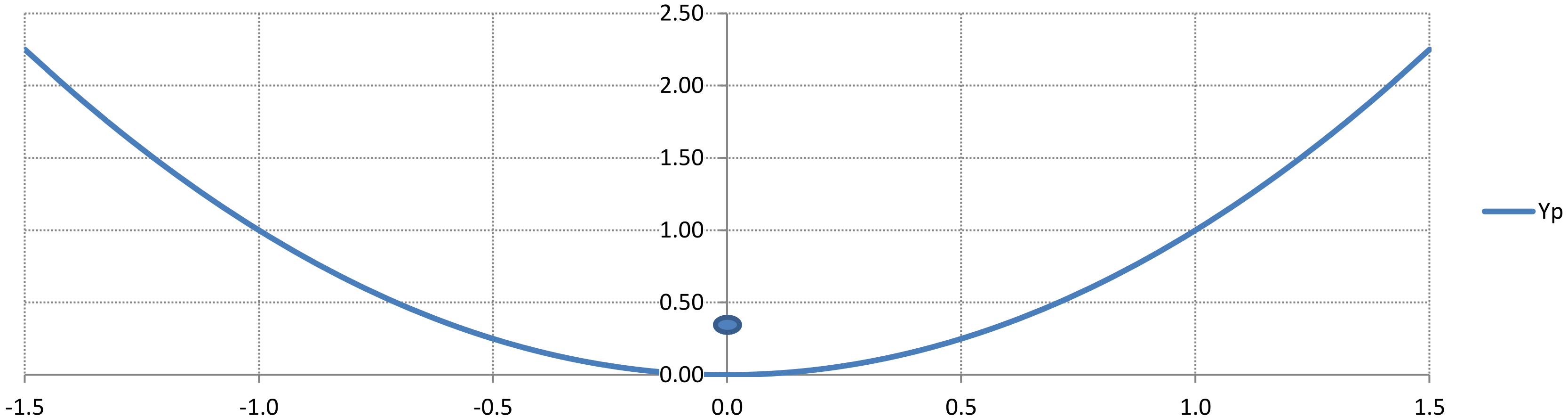


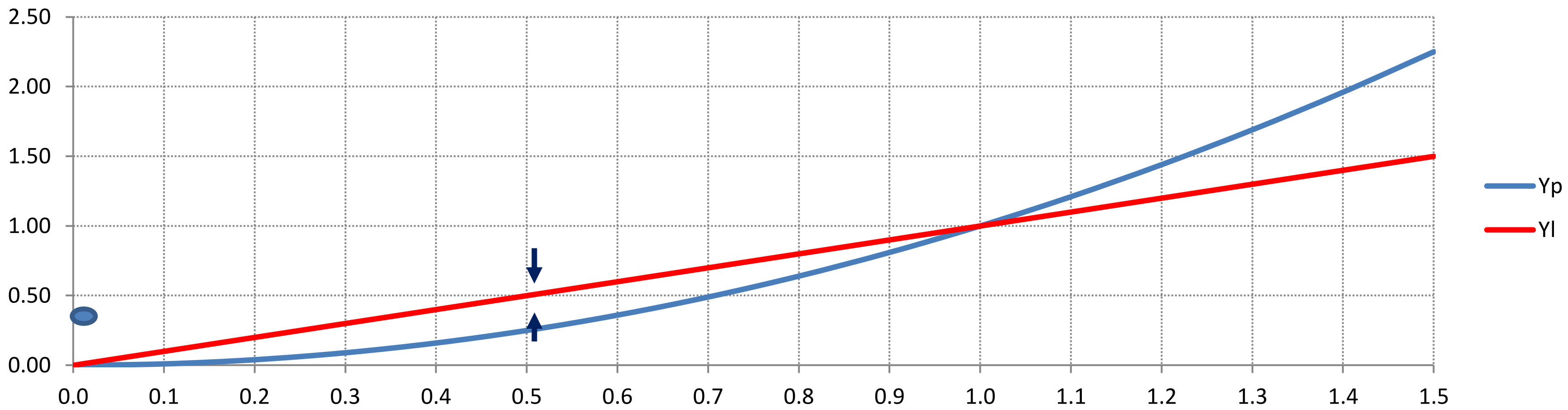
- ***AXIAL FEED HORN BLOCKS DISH APERTURE***
- ***AXIAL FEED MATCH IS AFFECTED BY DISH REFLECTION***
- ***OFFSET FEED HAS A LARGER EFFECTIVE f/D***

Assumptions

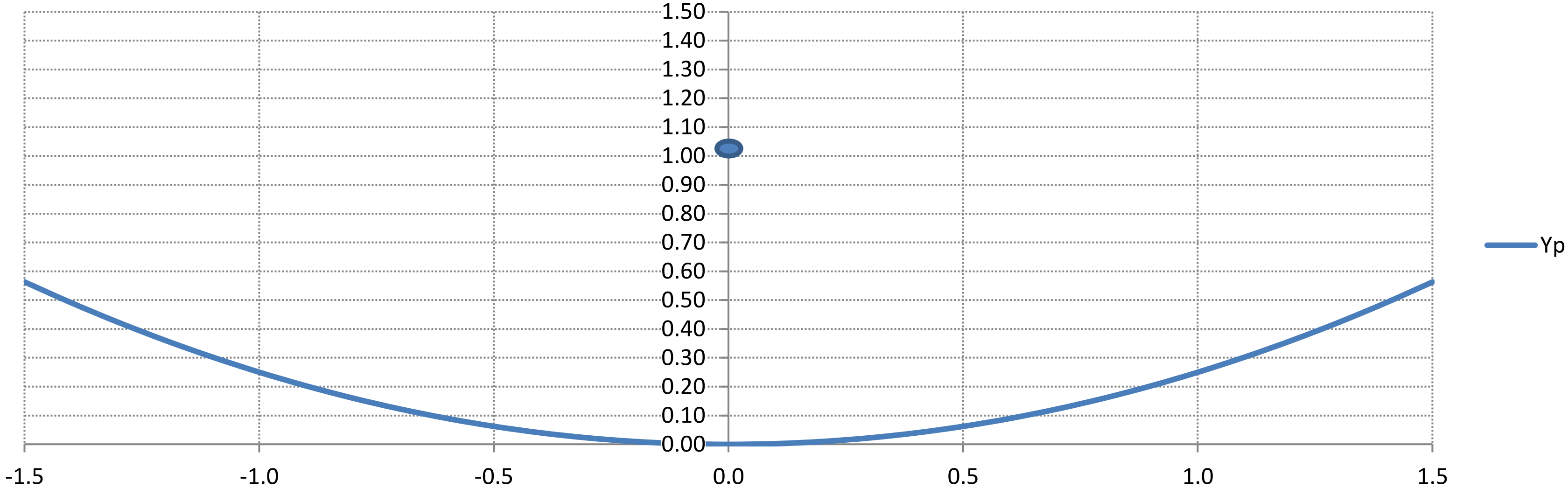
- Analysis based on measurements along vertical cross-section of dish
- Bottom of dish is origin of full parabola
- Two measurements determine solution:
  - Vertical width of dish
  - Depth of dish at center

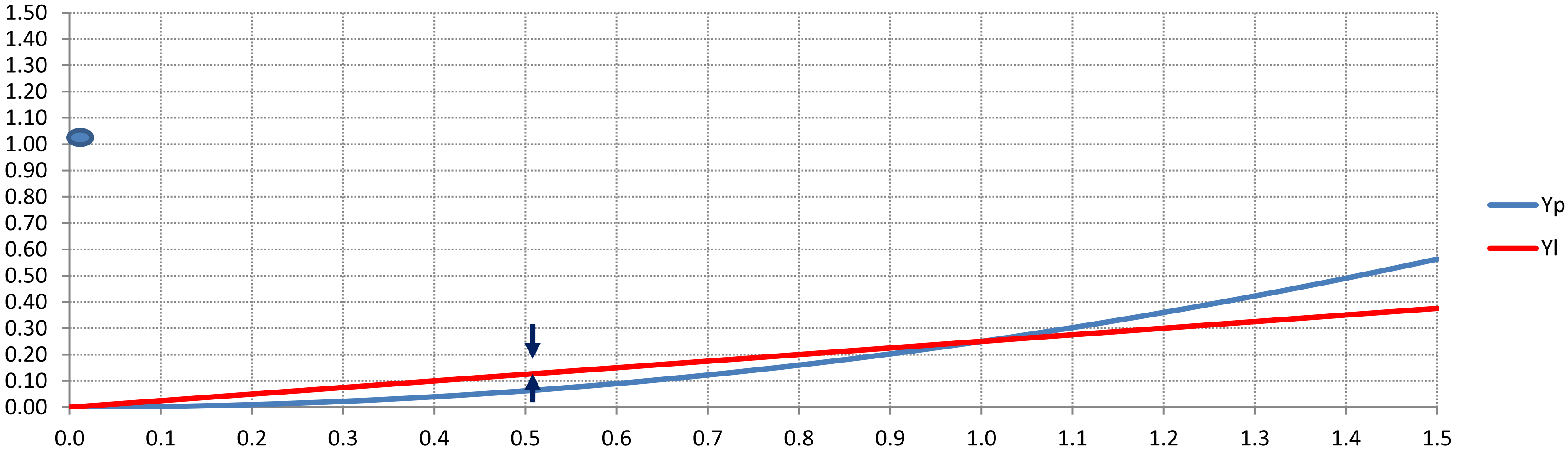
Deep-Dish Example



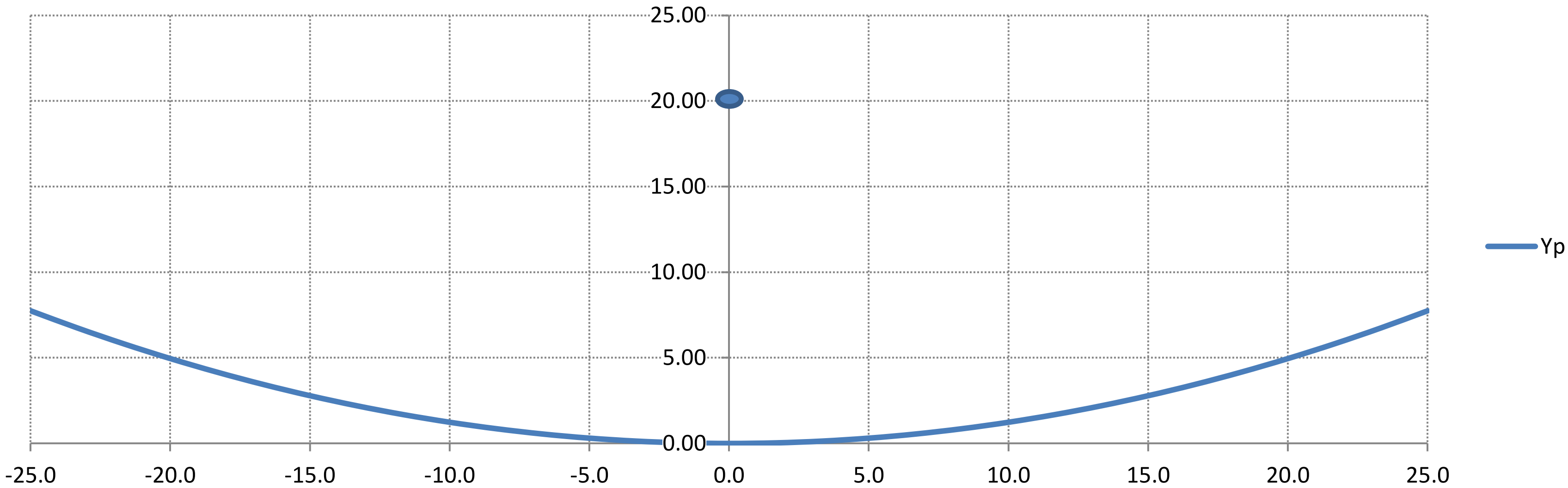


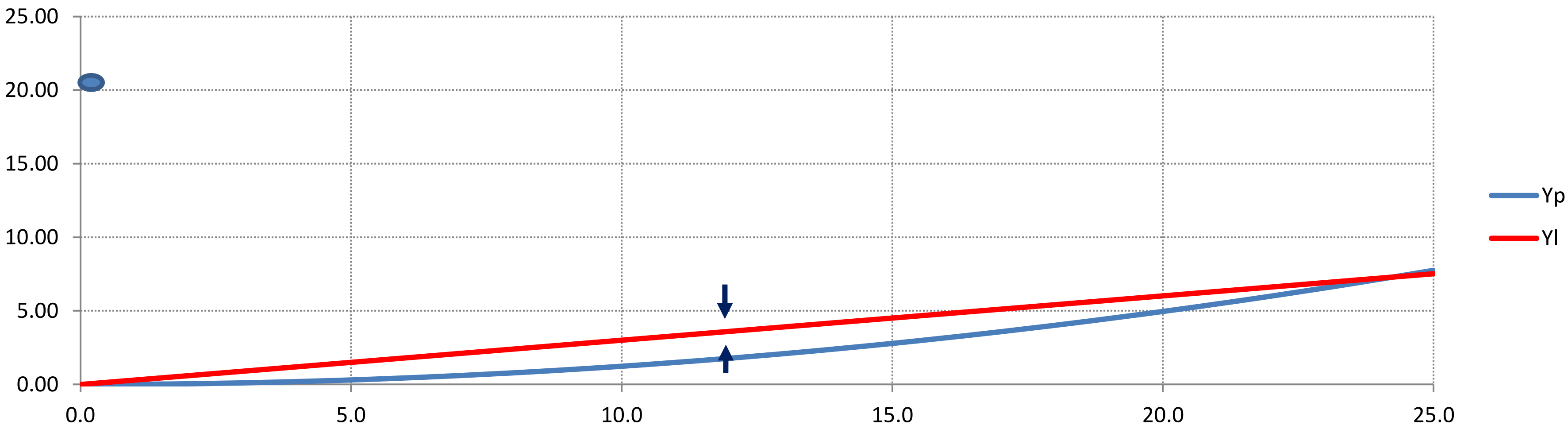
Shallow-Dish Example



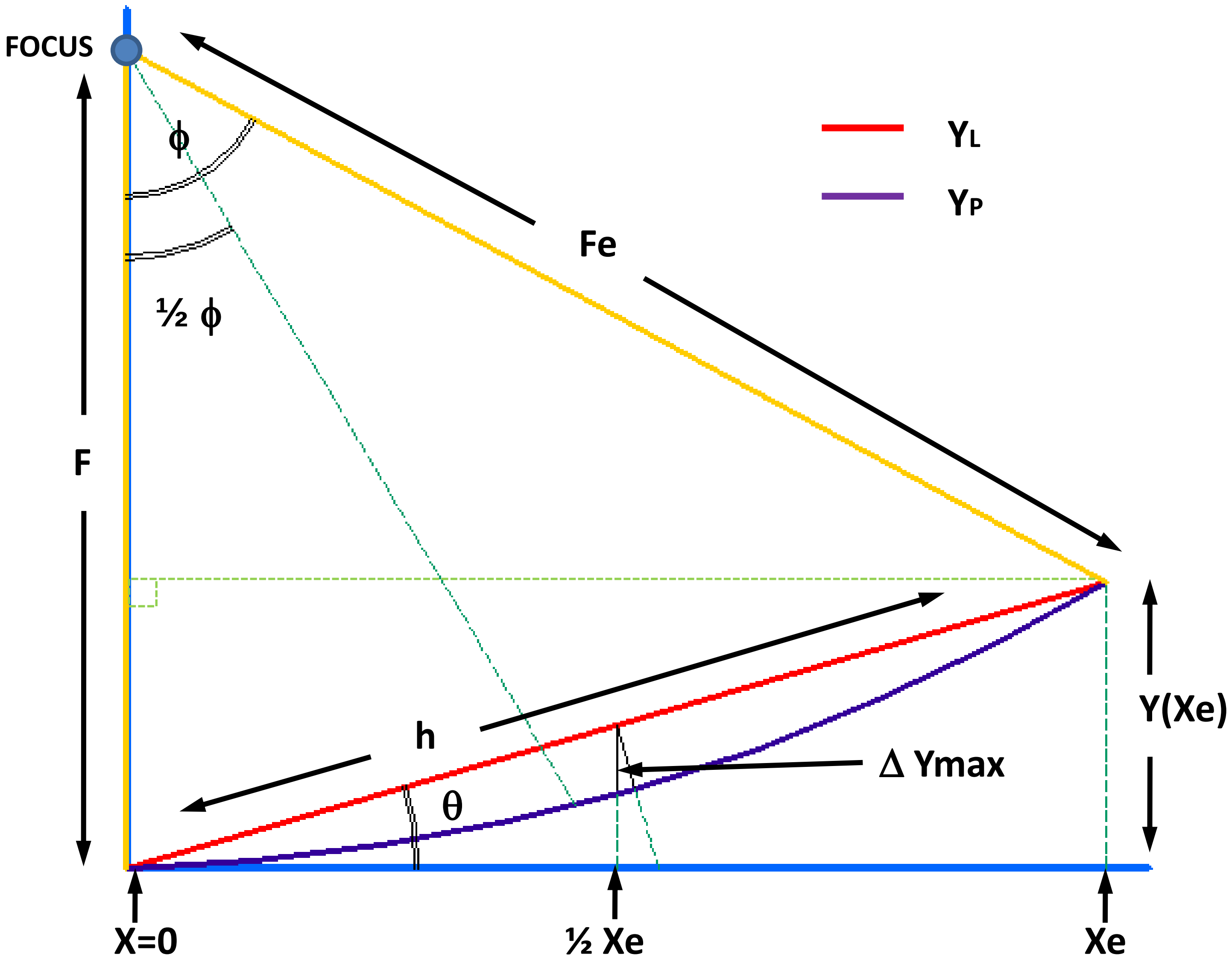


HughesNet Dish





Offset-Feed Geometry



Analysis

Linear equation (vertical width): $y_L = ax$

Parabolic equation (offset-dish): $y_P = b\,x^2$

Equivalence at top end: $y_L(x_e) = y_P(x_e)$

$$a\,x_e = b\,x_e^2$$

$$x_e = \frac{a}{b}$$

Depth of dish: $y_\Delta = y_L - y_P \quad = a\,x - b\,x^2$

Find maximum: $\frac{d}{d_x} y_\Delta = a - 2bx \quad \equiv 0$ (set to zero and solve)

$$\max y_\Delta \xrightarrow{yields} x = \frac{a}{2\,b} \quad = \frac{x_e}{2}$$

$$\max y_\Delta \quad = \frac{a x_e}{2} - \frac{b {x_e}^2}{4} \qquad = \frac{y_L(x_e)}{2} - \frac{y_P(x_e)}{4}$$

Define y_{MAX}: $y_{max} = y_L(x_e) = y_P(x_e)$

$$\max y_\Delta = \frac{y_{max}}{2} - \frac{y_{max}}{4} = \frac{y_{max}}{4}$$

From measured $\max y_\Delta$: $y_{max} = 4\,(\max y_\Delta)$, this always occurs at $x = x_e$

Tilt angle: $\theta = \sin^{-1}\left[\frac{y_{max}}{h}\right]$, h is vertical width of dish

Analysis

Find axial focus: $(F - y_{max})^2 + x_e^2 \approx [2F - (F - y_{max})]^2$ *(assumes $\Phi \approx 40°$)*

$x_e^2 = 4\,F\,y_{max}$

$F = \frac{1}{4\,b}$

$y_{max} = b\,x_e^2 \xrightarrow{yields} b = \frac{y_{max}}{x_e^2}$

$x_e = \sqrt{h^2 - y_{max}^2}$

$b = \frac{y_{max}}{(h^2 - y_{max}^2)}$

$F = \frac{h^2 - y_{max}^2}{4\,y_{max}} = \frac{h^2 - 16(\max y_\Delta)^2}{16\,max y_\Delta} = \frac{h^2}{16\max y_\Delta} - \max y_\Delta$

Top end focal distance: $F_e = \sqrt{x_e^2 + (F - y_{max})^2}$

Horn pointing angle: $\psi = \frac{\phi}{2} = \frac{1}{2}\cos^{-1}\left[\frac{F - y_{max}}{F_e}\right]$

Effective f/D : $\frac{\cot(\psi)}{2}$

HughesNet Dish Solution

MEASUREMENTS

$$\max y_{\Delta} = 1.8"$$

$$h = 25.2"$$

SOLUTION

$$F = \frac{25.2^2}{16(1.8)} - 1.8 = 20.25"$$

$$y_{max} = 4(1.8) = 7.2"$$

$$\theta = \sin^{-1}\left({}^{7.2}/_{25.2}\right) = 16.6° \quad \text{(tilt)}$$

$$x_e = 25.2 \cos(16.6) = 24.15"$$

$$F_e = \sqrt{24.15^2 + (20.25 - 7.2)^2} = 27.45"$$

$$\psi = \frac{\phi}{2} = \frac{1}{2}\cos^{-1}\left[\frac{20.25-7.2}{27.45}\right] = 30.8° \quad \text{horn pointing angle}$$

$$effective\ \frac{f}{D} = \frac{\cot(30.8)}{2} = 0.839$$