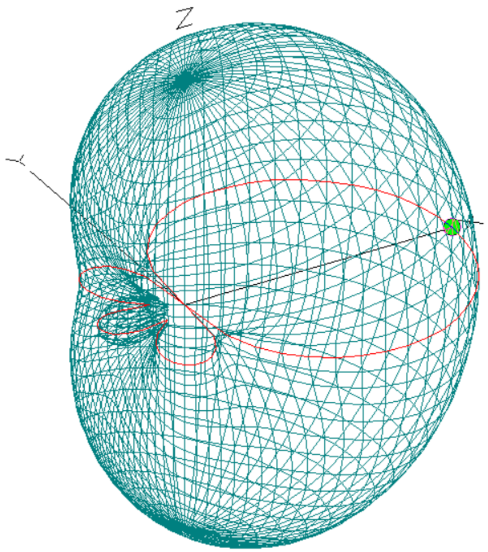
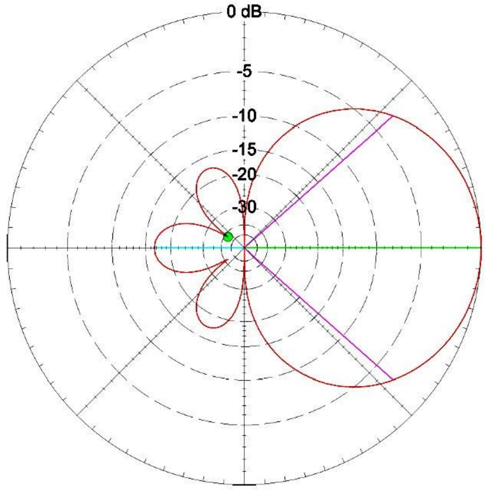
LINEAR ARRAYS

AN OVERVIEW





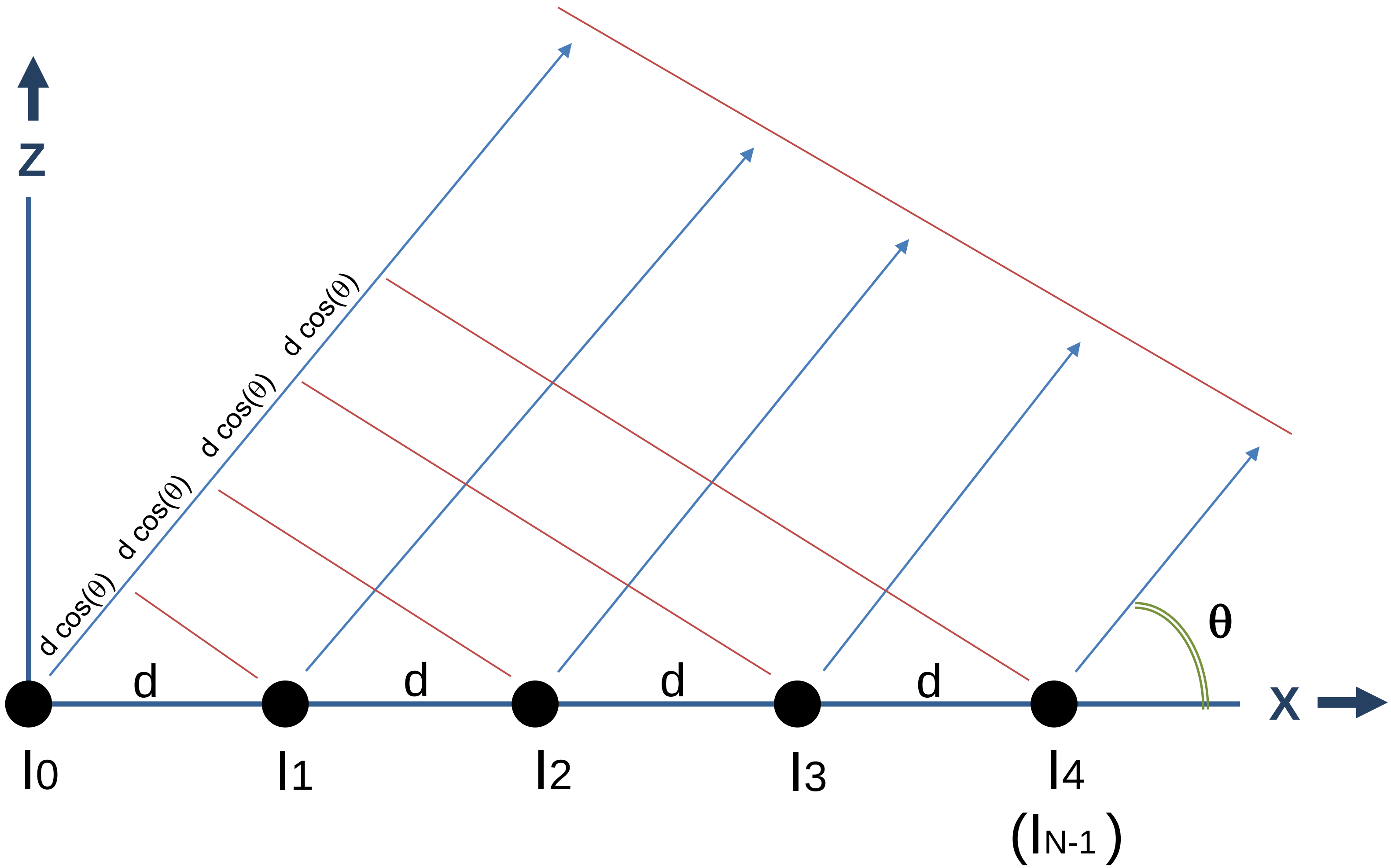
INTRO

- This presentation is intended for technical hams.
- One goal is to keep “out of the weeds” of excessive math by use of visuals to communicate the ideas. Some math is unavoidable.
- Another goal is to give insight into how linear arrays work and how they are designed.

OVERVIEW

- Element current causes radiation
- Element to element spacing and phase shift sets direction of maximum signal
- We will look at several types of linear arrays:
  - **UNIFORM:** equal amplitude drive to each element
  - **BINOMIAL:** element drive amplitudes proportional to polynomial coefficients (from binomial factors)
  - **SCHELKUNOFF:** superior gain and main to side lobe ratio from equally spaced nulls
  - **YAGI-UDA:** use of one driven element and mutual impedance to couple drive to parasitic elements

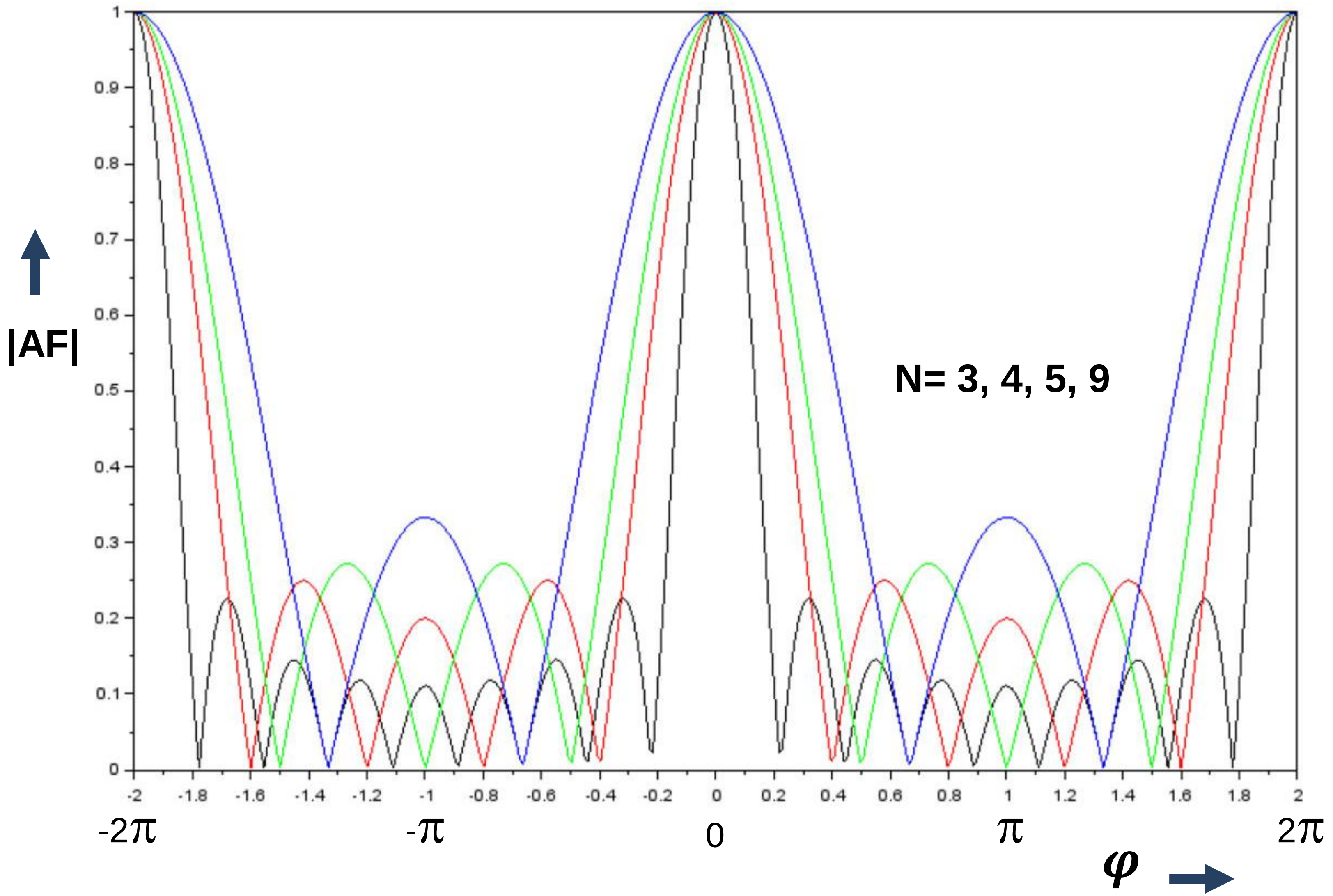
SIGNALS MUST ADD IN PHASE



UNIFORM ARRAY

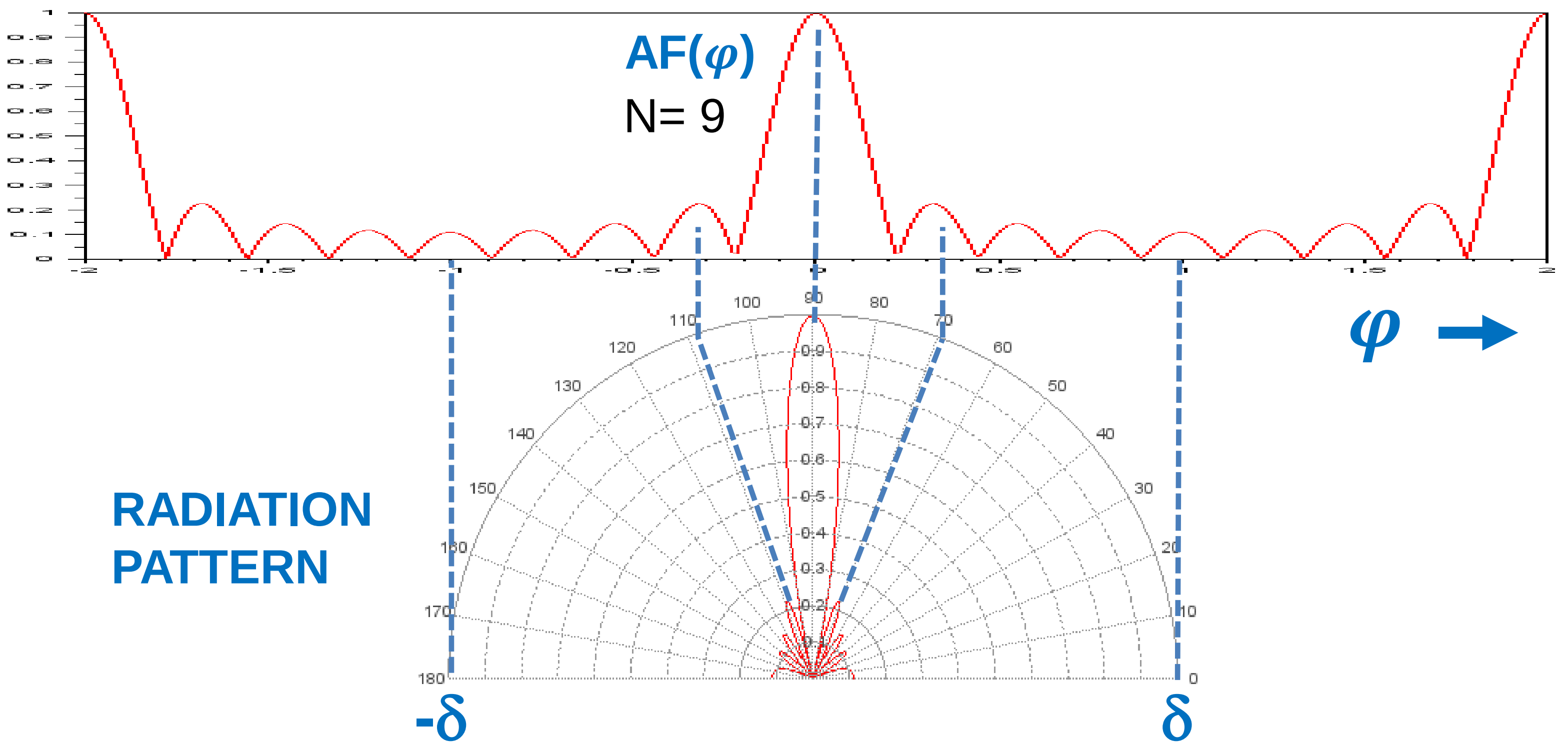
- Equal element spacing d
- Propagation delay phase shift from element to element is: $\frac{2\pi}{\lambda} d$ (radians). Let's call this δ .
- Element to element drive is same amplitude but phase shifted by same amount: α .
- Radiation pattern can be expressed in terms of an array factor (AF).
- AF = $I_0 + I_1 e^{j\delta \cos(\theta)+\alpha} + I_2 e^{j2(\delta \cos(\theta)+\alpha)} + I_3 e^{j3(\delta \cos(\theta)+\alpha)} \ldots\ldots +$ $I_{N-1} e^{j(N-1)(\delta \cos(\theta)+\alpha)}$
- Uniform drive: $I_0 = I_1 = I_2 = I_3 \ldots\ldots = I_{N-1}$ (hence the name)
- Substitute: $\varphi = \delta \cos(\theta) + \alpha$ and rewrite AF
- AF $= \sum_{n=0}^{N-1} A_0 e^{jn\varphi}$

UNIFORM ARRAY - ARRAY FACTOR



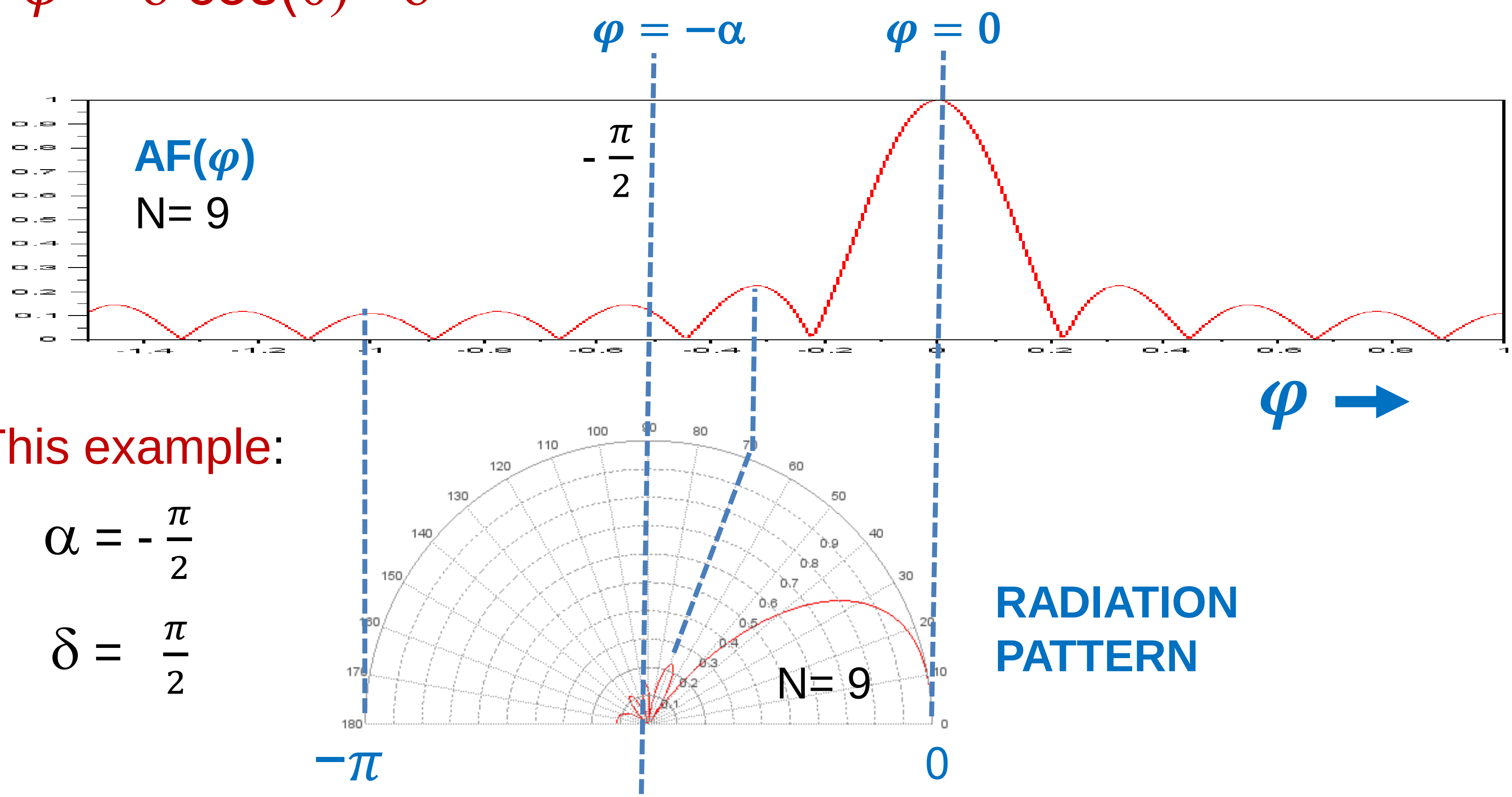
SO WHAT ?

- The same AF(φ) applies to all uniform arrays.
- It is periodic with 2π period repetition.
- The practical details are defined in $\varphi = \delta \cos(\theta)+\alpha$
- Broadside array: $\alpha=0$, $\varphi = \delta \cos(\theta)$



SO WHAT ?

- The practical details are defined in $\varphi = \delta \cos(\theta) + \alpha$
- Endfire array: $\alpha = -\delta$ (or $+\delta$).
- $\varphi = \delta \cos(\theta) - \delta$

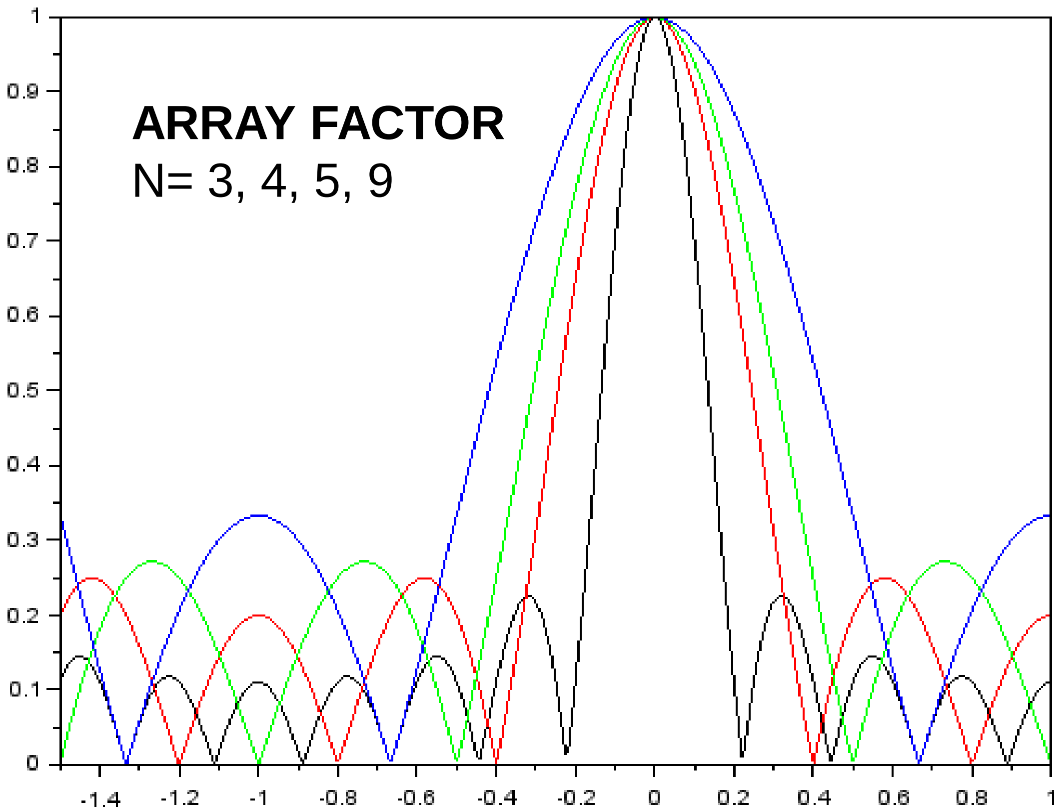


This example:

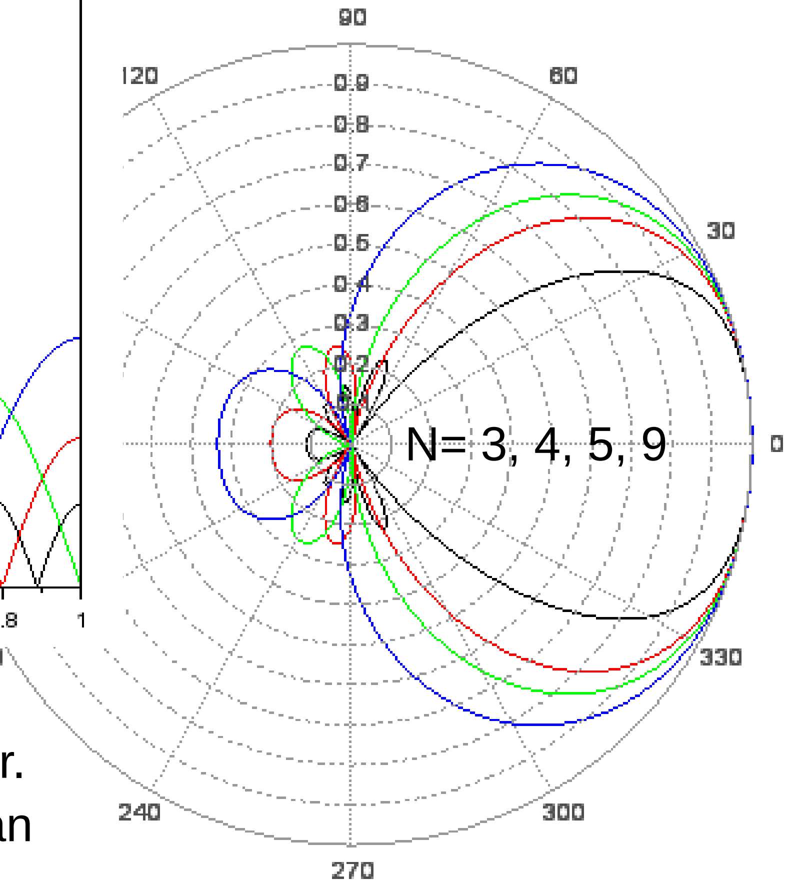
$\alpha = -\frac{\pi}{2}$

$\delta = \frac{\pi}{2}$

UNIFORM ARRAY: EQUAL AMPLITUDE DRIVE



RADIATION PATTERN



- Selection of α and δ allow the radiation pattern to be formed from the array factor.
- Main lobe, side lobe and null bearings can be set by design.

BINOMIAL ARRAY

- Equal element spacing d
- Element spacing propagation phase: $\delta = \frac{2\pi}{\lambda} d$ (radians).
- Element to element drive is phase shifted by same amount: α
- Current drive to elements is *NOT* uniform ←
- AF $=\sum_{n=0}^{N-1} A_n e^{jn\varphi}$
- The current drive for the n^th element is A_n .

BINOMIAL ARRAY

- AF =$\sum_{n=0}^{N-1} A_n e^{jn\varphi}$
- AF = A_0 + $A_1 e^{j\varphi}$ + $A_2 e^{j2\varphi}$. . . . + $A_{N-1n} e^{j(N-1)\varphi}$
- This looks like a polynomial if we let $e^{j\varphi}$ be represented as x. Then the AF becomes:
- AF =$\sum_{n=0}^{N-1} A_n x^n$, clearly a polynomial.
- A polynomial can be factored:

  AF =$A_{N-1}\ (x_1 - x)\ (x_2 - x)\ .\ .\ .\ .\ (x_{N-1} - x)$
- A special case is the binomial: AF =$(1 + x)^N$
- So we can write: AF = $(1+\ e^{j\varphi})^N$, *note: nulls are* $\pm\pi$

BINOMIAL ARRAY

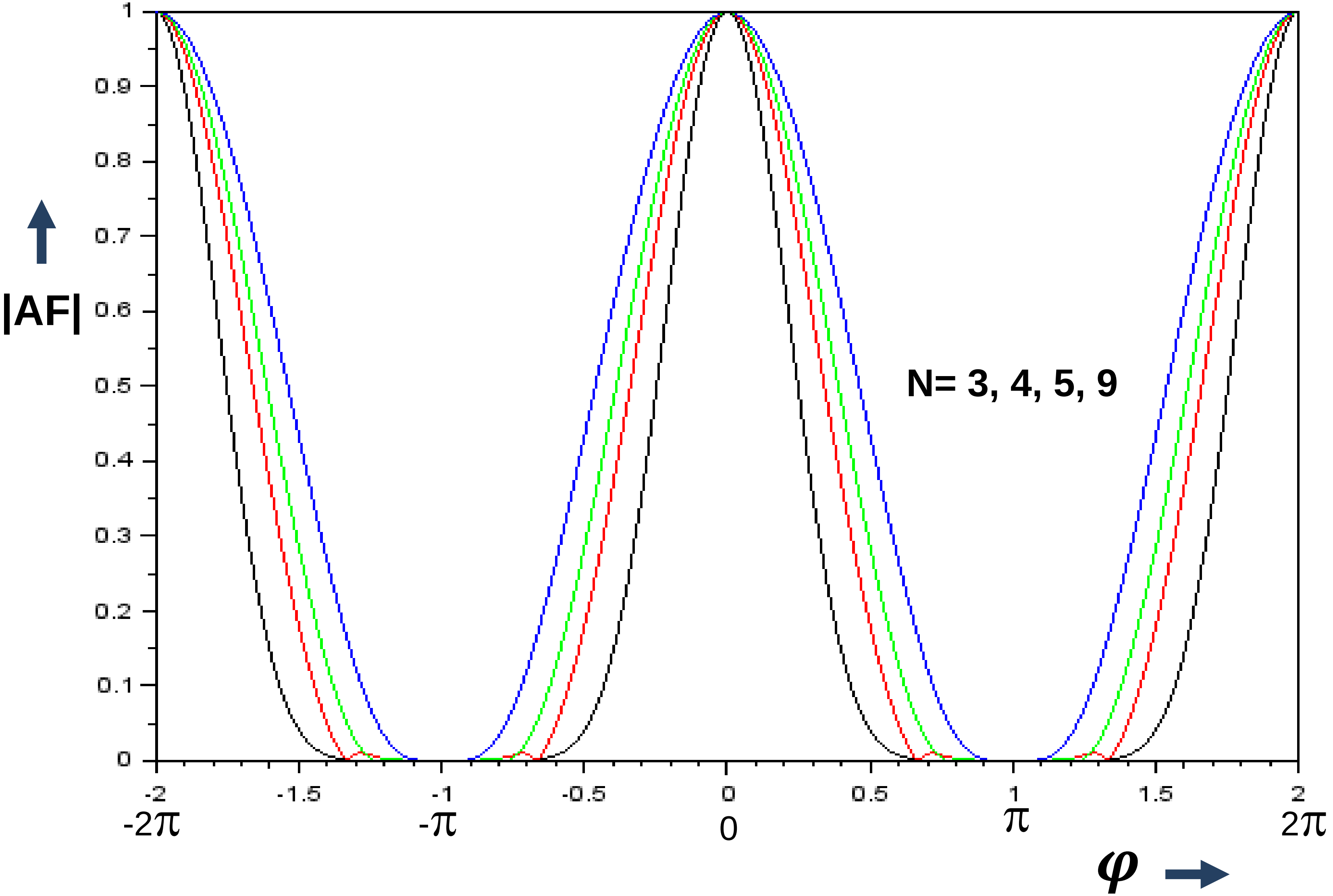
- The coefficients (A_n) of the binomial polynomial are well known.

- Pascal's triangle shows these coefficients:

N	BINOMIAL COEFFICIENTS																		
0										1									
1									1		1								
2								1		2		1							
3							1		3		3		1						
4						1		4		6		4		1					
5					1		5		10		10		5		1				
6				1		6		15		20		15		6		1			
7			1		7		21		35		35		21		7		1		
8		1		8		28		56		70		56		28		8		1	
9	1		9		36		84		126		126		84		36		9		1

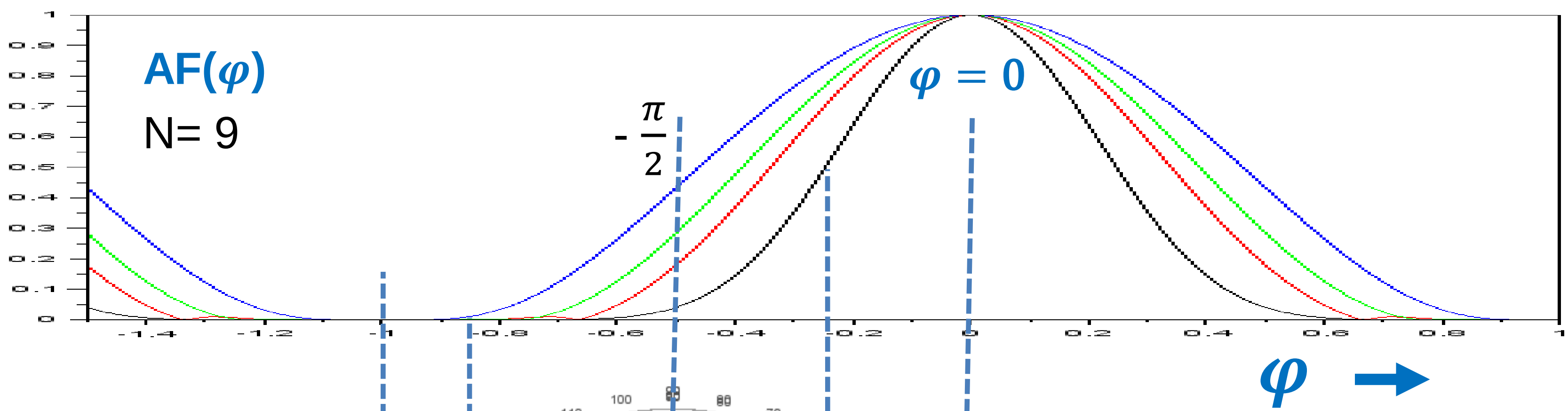
- The A_n coefficients represent the relative element currents.

BINOMIAL ARRAY - ARRAY FACTOR



BINOMIAL ENDFIRE ARRAY

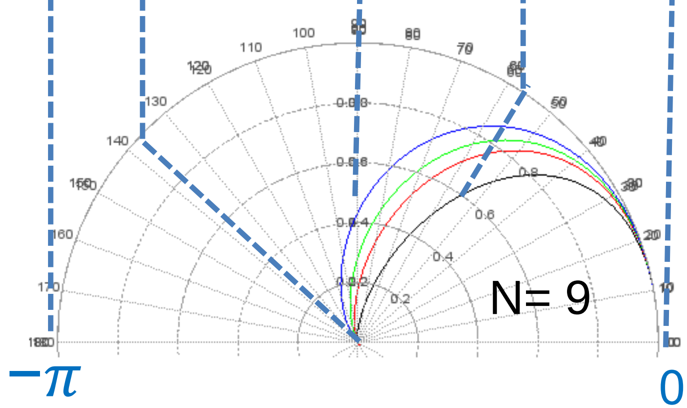
- Endfire array: α=-δ (or +δ).
- $\varphi = \delta \cos(\theta) - \delta$



This example:

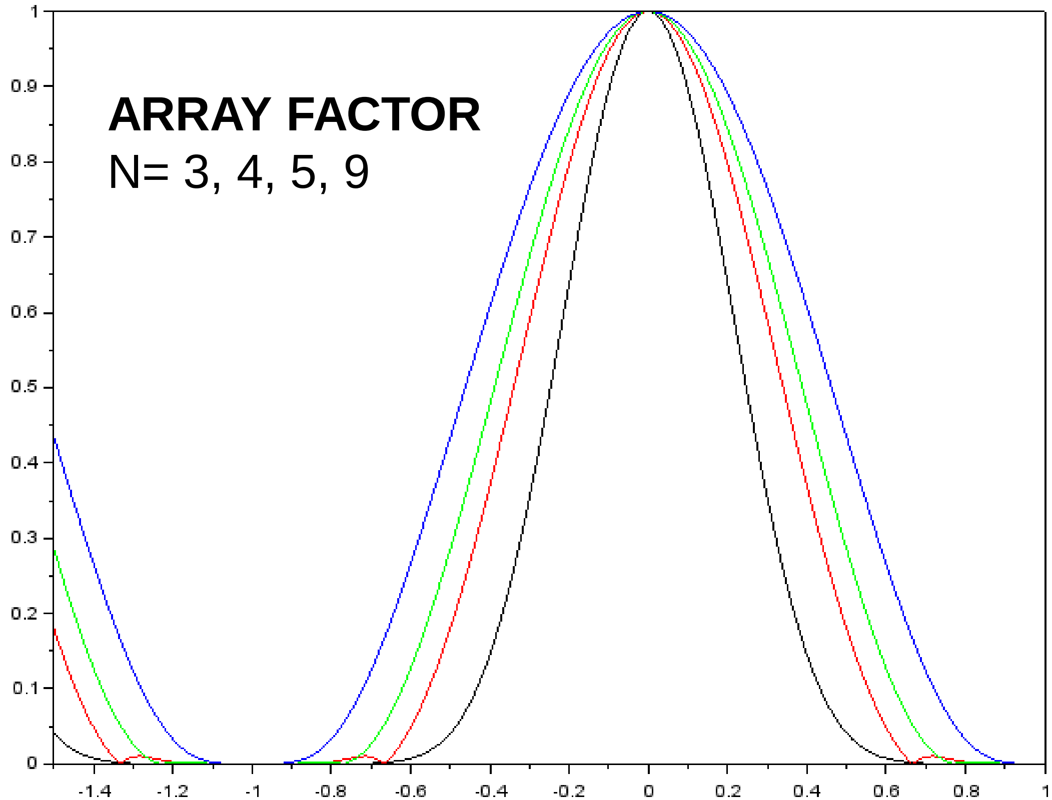
$$\alpha = -\frac{\pi}{2}$$

$$\delta = \frac{\pi}{2}$$

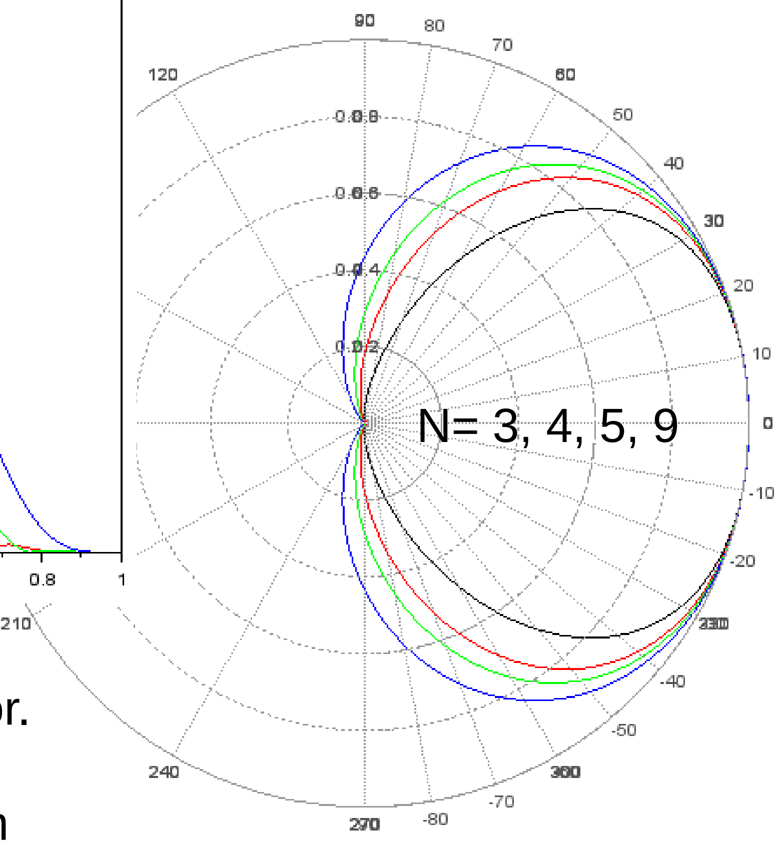


RADIATION PATTERN

BINOMIAL ARRAY: TAPERED AMPLITUDE DRIVE



RADIATION PATTERN



- Selection of α and δ allow the radiation pattern to be formed from the array factor.
- Main lobe bearing can be set by design.
- No side lobes and less gain than uniform array.

SO WHAT ?

- If a binomial array provides less gain than a uniform array, why is it important?
- The binomial array shows the representation of the AF as a polynomial.
- Other polynomials might provide better gain. For example, a Chebyshev polynomial can yield equal side lobe levels.
- Side lobes mean the array has nulls.
- The key to improved gain is low side lobe levels and equally spaced nulls.

HANSEN and WOODYARD

- Both previous endfire examples had $\frac{\lambda}{4}$ spacing and $\left|\frac{\pi}{2}\right|$ phase offset.
- Hansen and Woodyard showed that the optimum directivity occurs at:
  - ✓ Spacing = $\left[\frac{N-1}{N}\right]\left[\frac{\lambda}{4}\right]$ and
  - ✓ Phase offset = $\left|\frac{\pi}{2}+\frac{2.94}{N}\right|$.
- The sidelobe levels will degrade somewhat from a $\frac{\lambda}{4}$ spacing and $\left|\frac{\pi}{2}\right|$ phase offset design.

SCHELKUNOFF ARRAY

- S.A Schelkunoff recognized that Array Factors for discrete arrays could be represented as polynomials.
- The expression of the AF polynomial in factored form shows where the nulls occur:

AF $= A_{N-1}\,(x_1 - x)\,(x_2 - x) \ldots . (x_{N-1} - x)$.

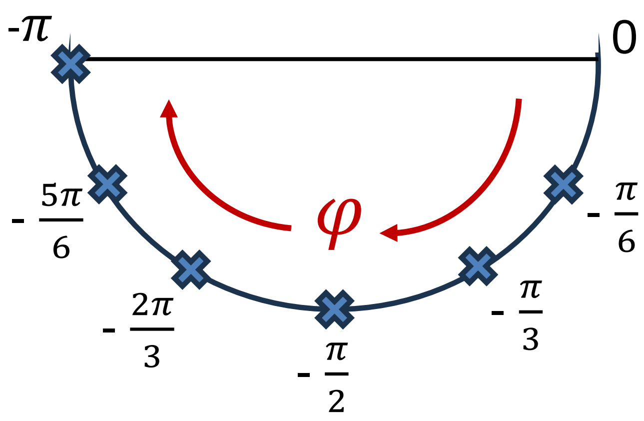
- The nulls are at $x = x_i$ for i between 1 and N-1. The nulls are called the roots of the polynomial.
- Since |AF| must be between 0 and 1, $|\, x_i \,| = 1$, for all i .
- This is consistent with $\boldsymbol{x = e^{j\varphi}}$. All values of x are on the unit circle.

SCHELKUNOFF ARRAY

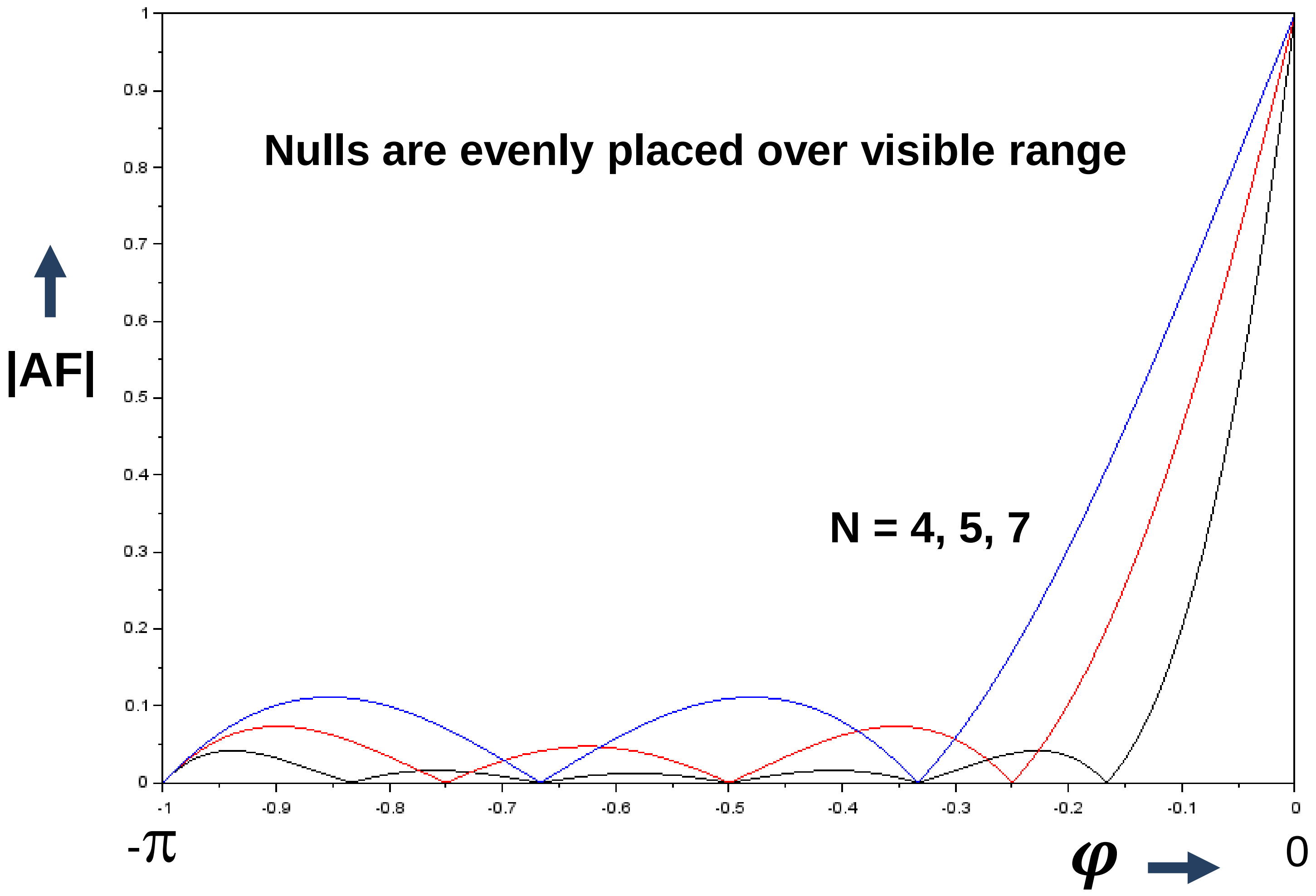
- The Schelkunoff array places nulls with equal spacing across the visable range of the AF.
- For example, let's consider a seven element endfire array:

  $\alpha = -\delta = -\frac{\pi}{2}$, so visible range of φ is 0 to $-\pi$ (-180°).

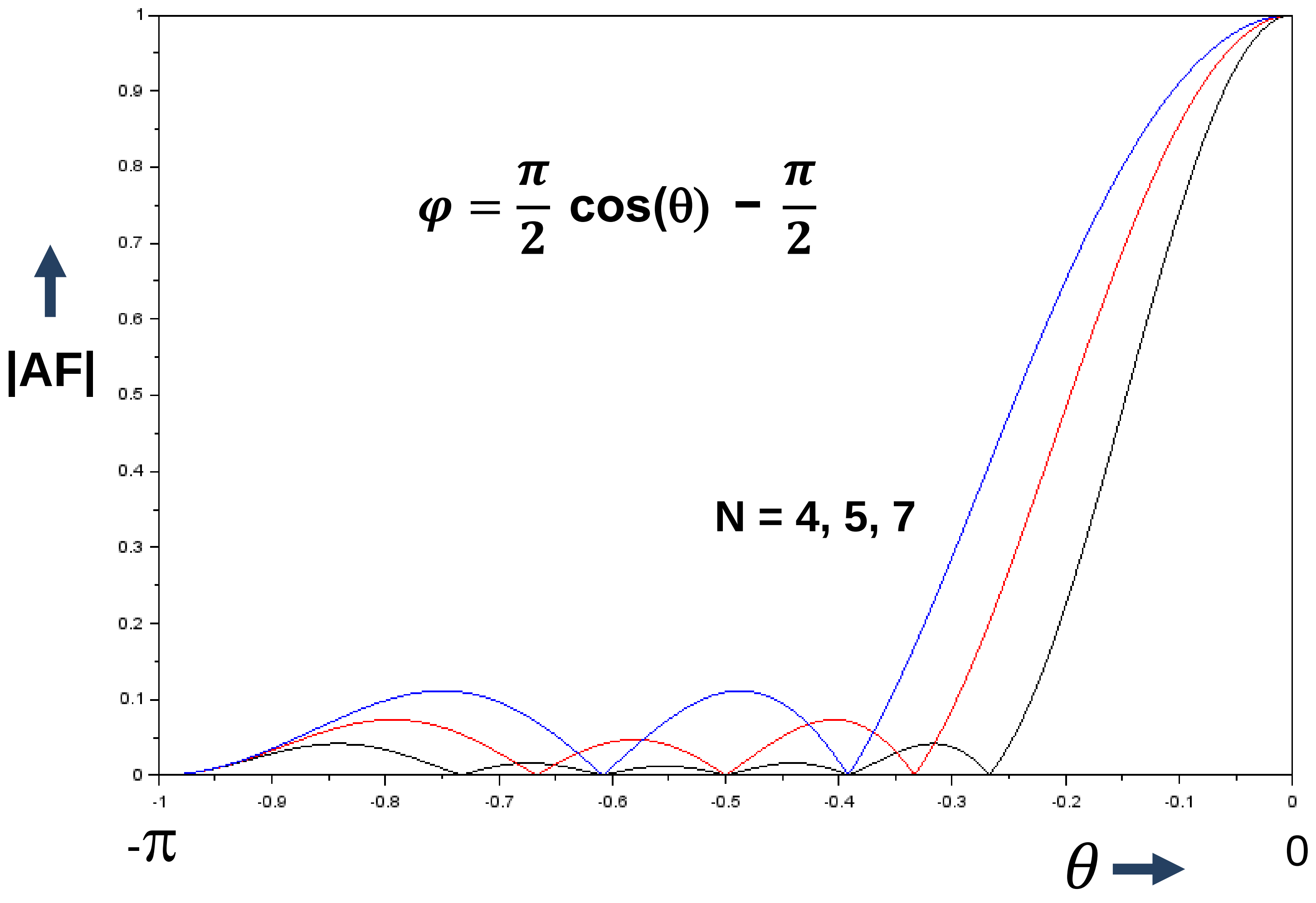
- The N-1 (6) nulls are places in 30° steps along the half circle from 0° to -180°. In radians, these are equal $\pi/6$ steps from 0 to $-\pi$.



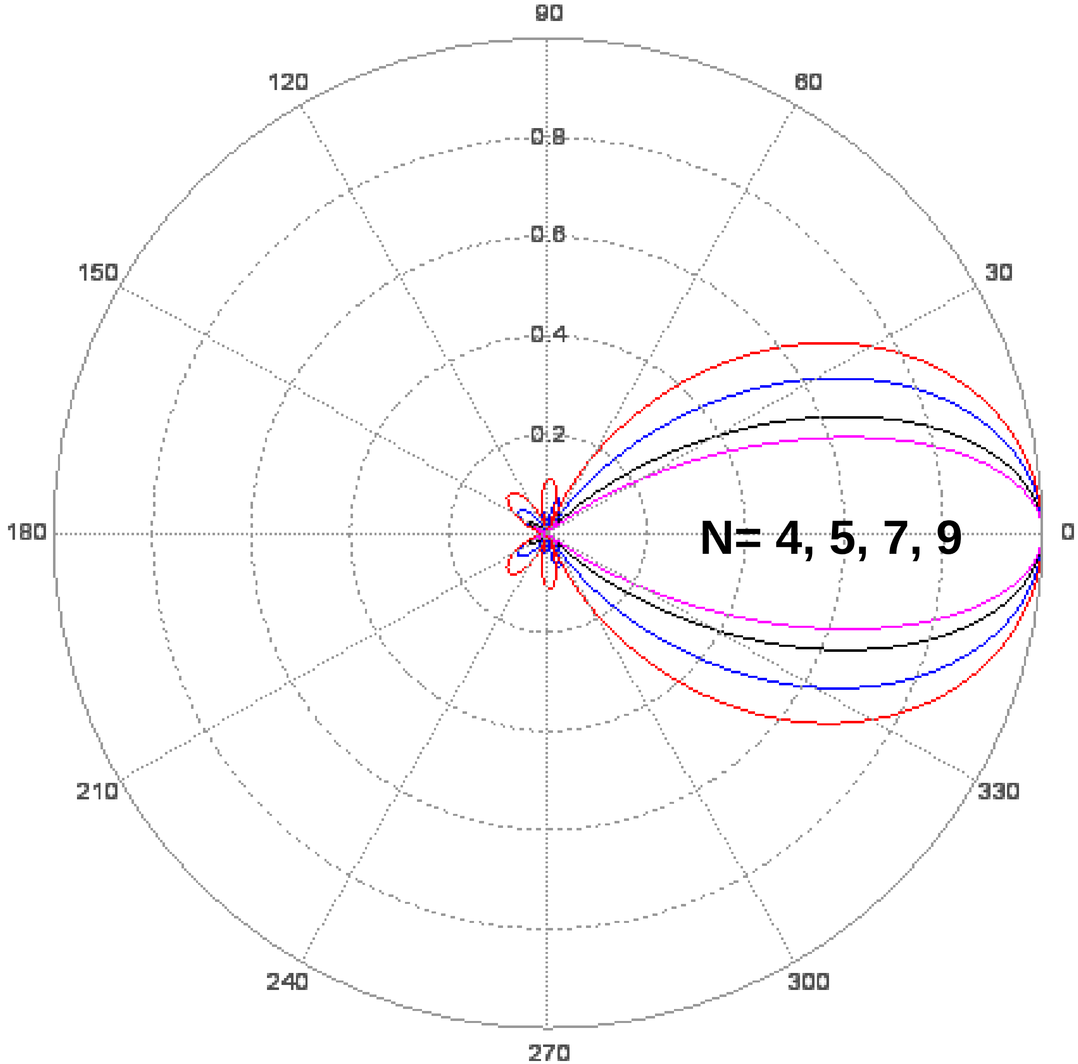
SCHELKUNOFF ARRAY - ARRAY FACTOR (φ)



SCHELKUNOFF ARRAY - ARRAY FACTOR (θ)



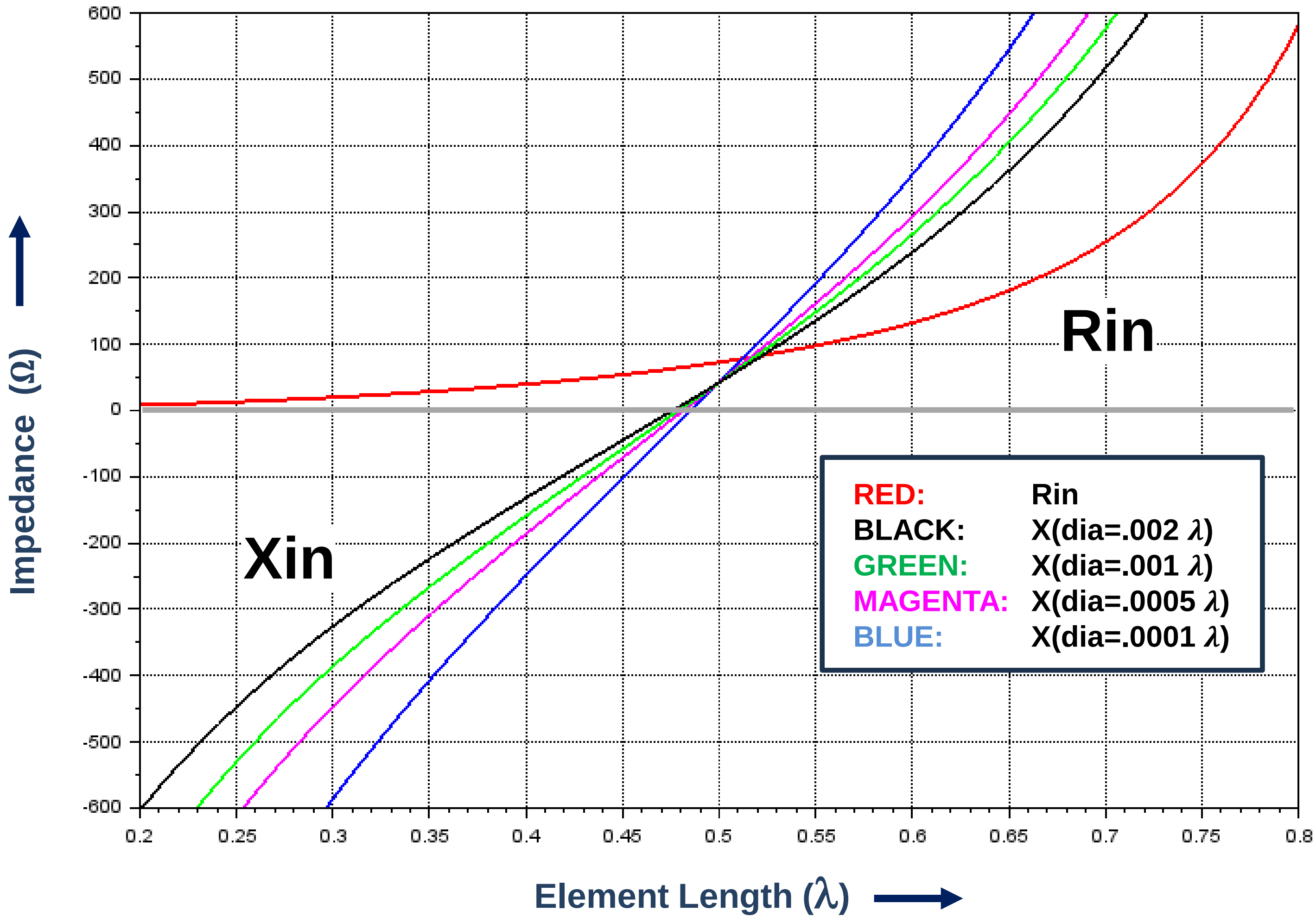
SCHELKUNOFF ARRAY - RADIATION PATTERN



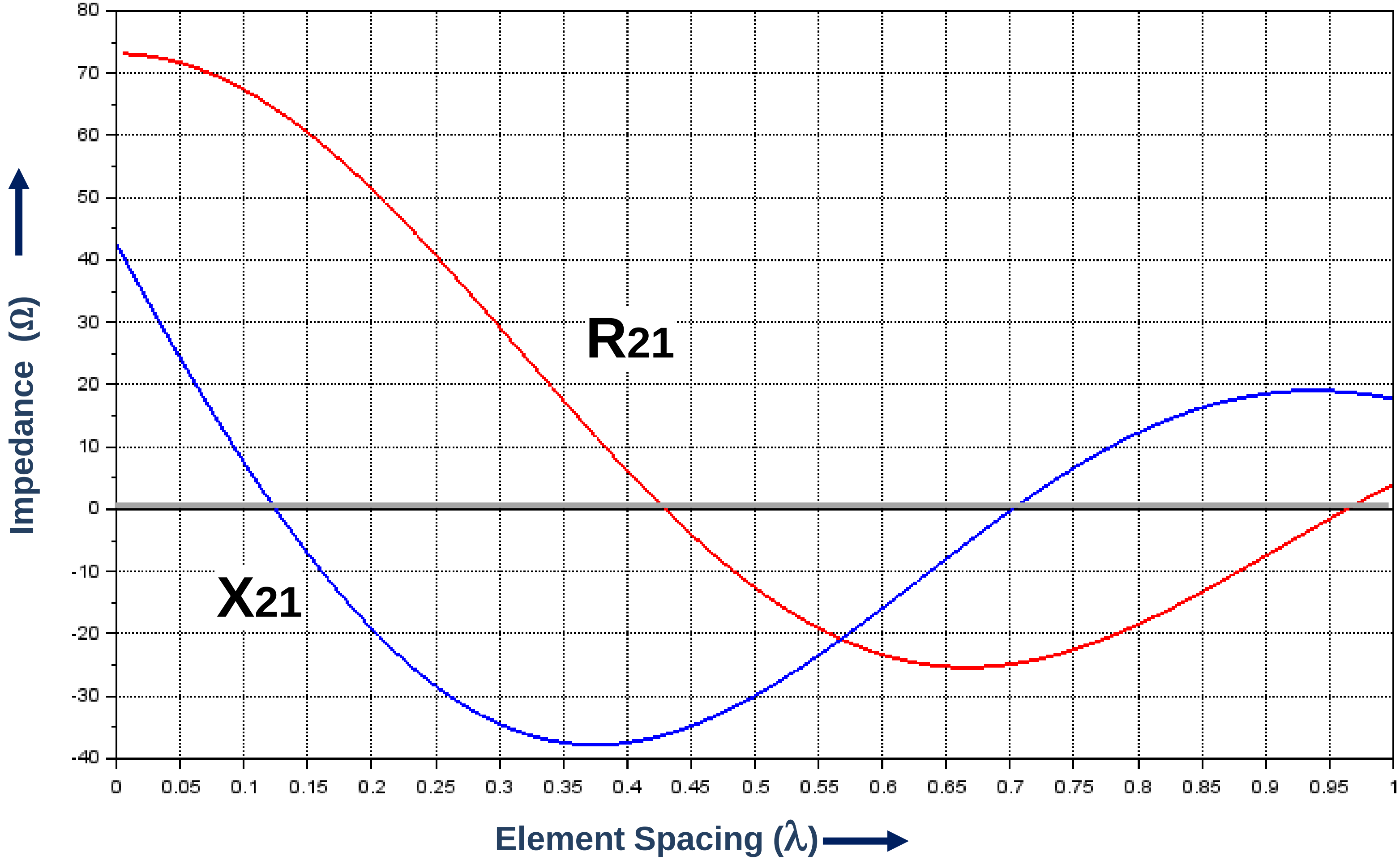
YAGI-UDA ARRAY

- Yagi-Uda arrays are endfire and usually have only one driven element.
- Currents in other elements are driven by coupling through mutual impedance with the driven element.
- The relative phase and magnitude of the passive element currents is controlled by element spacing and length.
- Shintaro Uda invented this array type in 1926. It should rightly be called an Uda Array

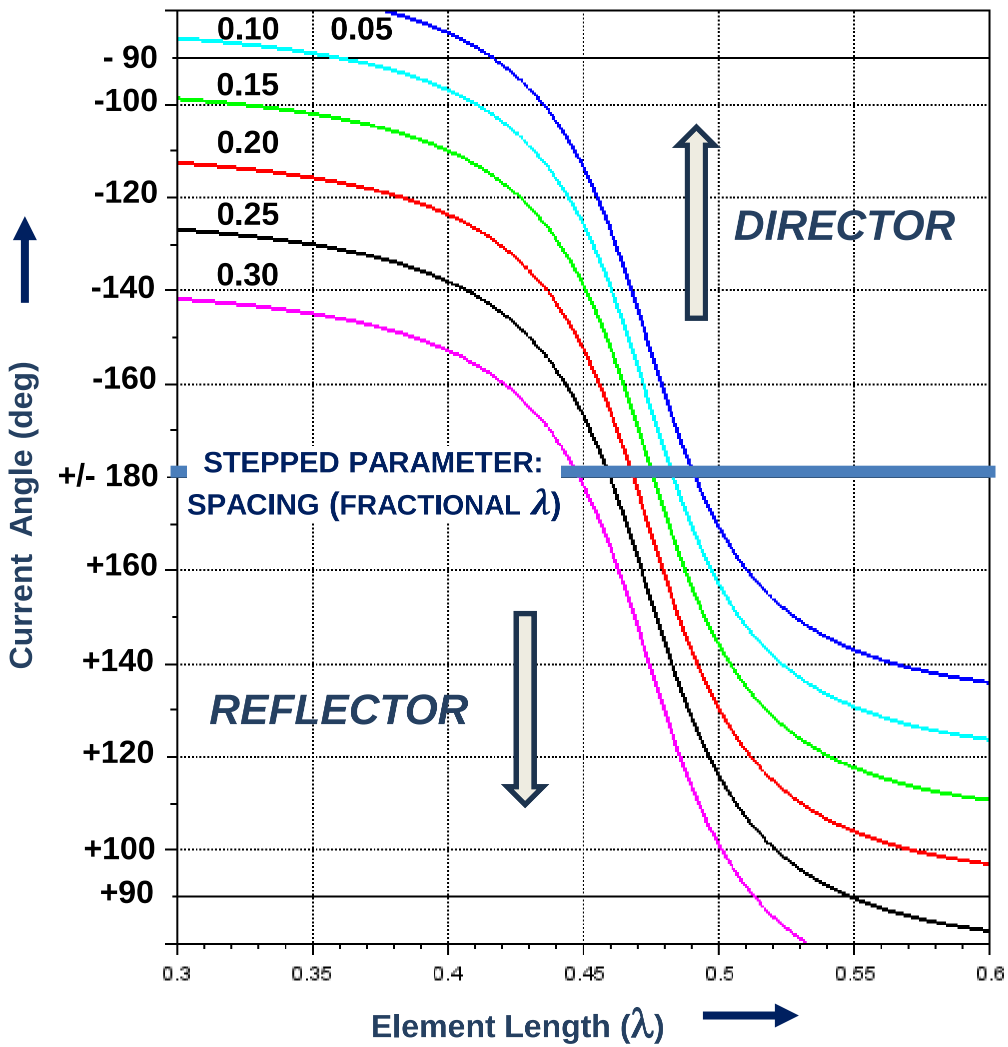
DIPOLE CENTER FEED IMPEDANCE



PASSIVE ELEMENT MUTUAL IMPEDANCE

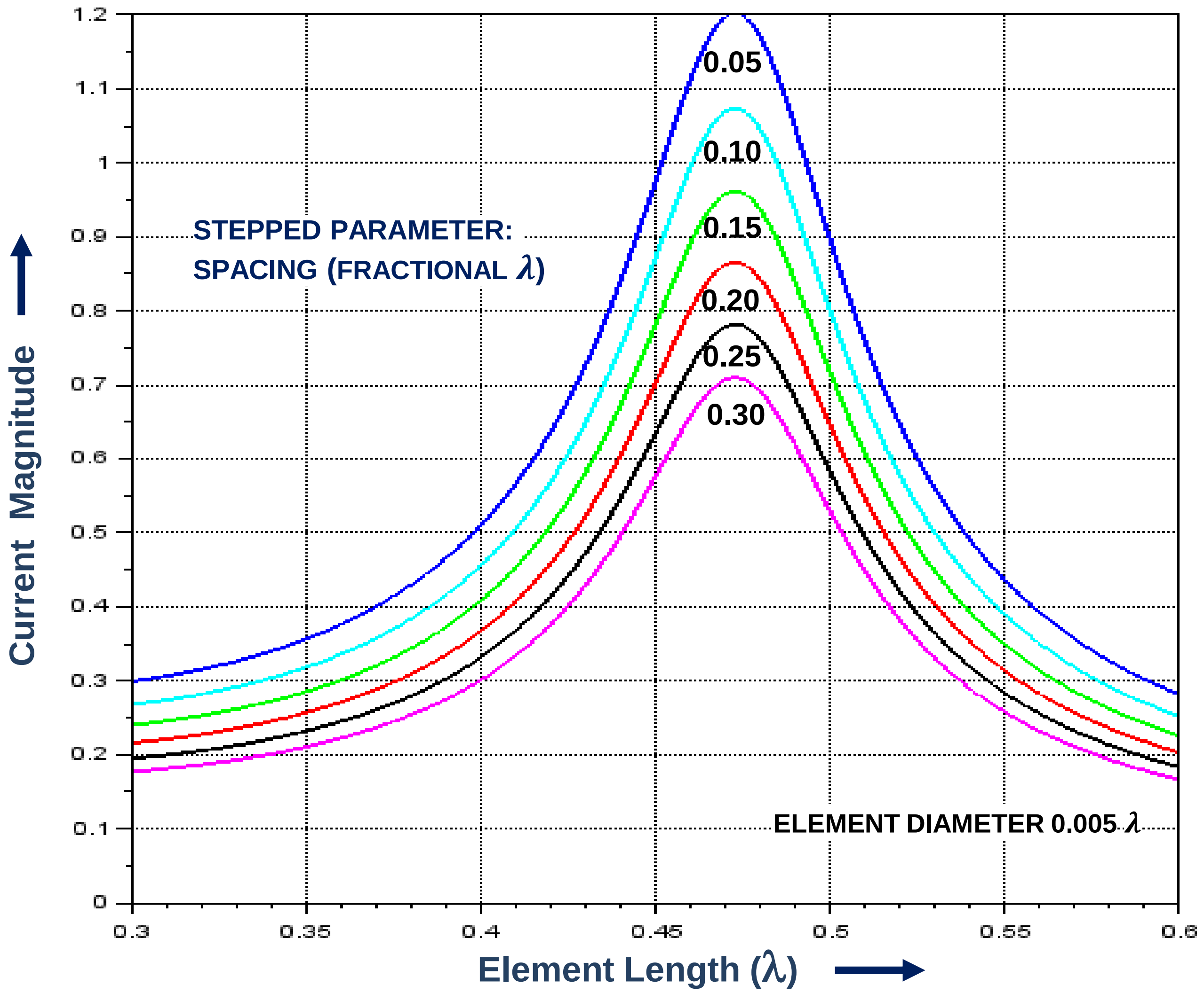


PASSIVE ELEMENT COUPLED CURRENT PHASE

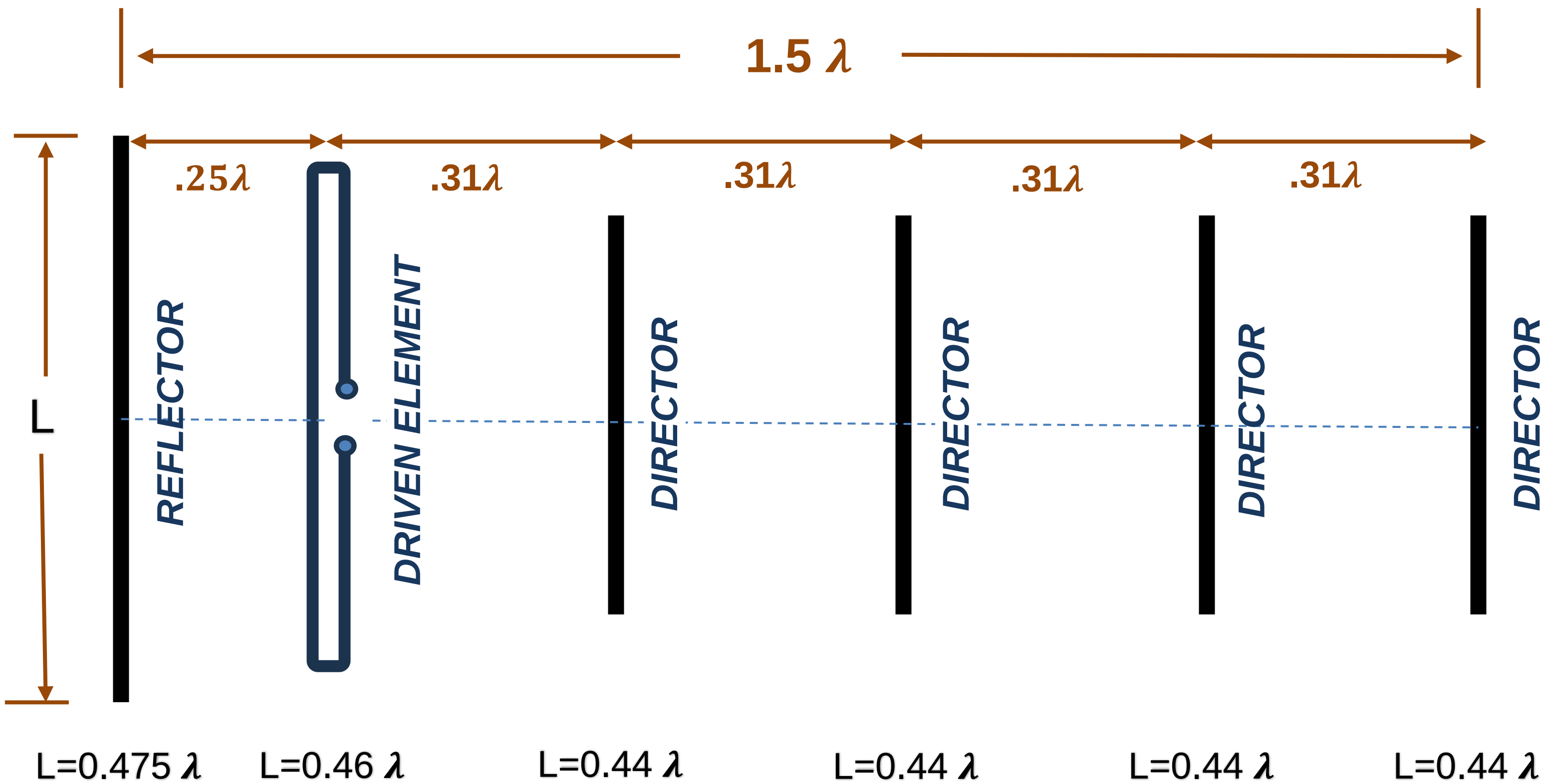


- Equal length, lightly coupled elements have currents 180° out of phase
- Tight coupling causes current phase to shift from 180°
- Coupled element length can make the phase shift greater than or less than 180°
- Current lag (- phase): director element
- Current lead (+phase): reflector element

PASSIVE ELEMENT COUPLED CURRENT MAGNITUDE



YAGI-UDA ARRAY - 6 ELEMENT EXAMPLE



SUMMARY

- Element current causes radiation.
- Array pattern is sum of signal from all elements.
- Amplitude and phase from each element is controlled by spacing and drive current.
- Array Factors provide a convenient tool for design.
- A selection of examples have been presented:
  - **Uniform** (equal drive currents)
  - **Binomial** (binomial coefficient drive currents)
  - **Schelkunoff** (equal spaced AF nulls)
  - **Yagi-Uda** (parasitic element drive through mutual impedance coupling)
- The arrays considered, all have elements placed along the x axis. The techniques discussed, can be applied simultaneously to elements along the other axes to create “curtain arrays”.